

Instructional Plan  
Abstract Level

Name of Math Skill/Concept: Addition of fractions with mixed numbers (Like denominators).

**Prerequisite Skills Needed:**

- 1.) Ability to add fractions with mixed numbers using concrete objects.
- 2.) Ability to add fractions with mixed numbers by drawing pictures.
- 3.) Familiarity with place value mats representing "ones" and "tens."

**Learning Objectives:**

- 1.) Add fractions with mixed numbers using a "fraction place value mat" and a fraction number line.
- 2.) Solve story problems and equations involving addition of fractions with mixed numbers using the FASTDRAW Strategy without drawing.

**Important Ideas for Implementing This Teaching Plan:**

1.) This plan describes the use of several cueing devices that will facilitate students' transition from drawing solutions to solving problems at the abstract level. These cueing devices include a fraction place value mat and fraction number lines. Some students will move quickly from adding fractions with mixed numbers using these cues to solving such problems without using these cues. Other students will benefit from their use for longer periods of time and the fading of use of these cues will need to be more systematically applied. Students may periodically need to use drawings to solve problems at the abstract level as a "back-up" strategy. This should not be discouraged, especially as they initially solve problems at this level. You can help students fade their use of drawings by setting goals for decreasing numbers of problems they use drawings for, intermixing problems you know they can solve without drawing with problems they will probably need to draw solutions for.

2.) This plan also describes how to solve story problems that involve addition of fractions with mixed numbers. Continuing to teach this skill using the FASTDRAW Strategy will help students with memory problems to "remember" the steps needed to solve such problems. The only difference between teaching FASTDRAW at the abstract level and the representational level is that you will teach students to apply abstract problem-solving methods during the "A" step in DRAW rather than drawing solutions.

3.) At the abstract level, it is important to systematically help students fade the use of cues such as fraction place value mats, fraction number lines, and FASTDRAW Strategy Cue Sheets so that eventually they solve problems without such cueing.

Instructional Phase 1: Initial Acquisition of Skill/Concept – Teacher Directed Instruction

**Teach Skill/Concept within Authentic Context**

*Description:*

Continue to link adding fractions with mixed numbers without drawing to meaningful contexts such as the Pizza Party example described in the Representational/Drawing and Concrete Level Instructional Plans. This is especially important as you teach students to solve story problems without drawing using the FASTDRAW Strategy. Additionally, make explicit links to drawing experiences when teaching the abstract process.

**Build Meaningful Student Connections**

*Purpose:* to assist students to build meaningful connections between what they know about adding fractions with mixed numbers by drawing pictures and adding fractions with mixed numbers without drawing.

Learning Objective1: Add fractions with mixed numbers using a “fraction place value mat” and a fraction number line.

*Materials:*

Teacher -

- a platform for writing/drawing so that all students can see (e.g. chalkboard/dry-erase board; chart paper; overhead projector.).
- the written objective: “add fractions with mixed numbers **without drawing.**” (\*Highlight the words “without drawing” for cueing purposes.).
- chalk/markers

*Description:*

1.) Link to students' prior knowledge of adding fractions with mixed numbers by drawing to adding fractions with mixed numbers without drawing.

For Example:

You now know how to solve story problems and equations that involve addition of fractions with mixed numbers by drawing pictures that represent concrete materials. Let's review how to add fractions with mixed numbers by drawing pictures. (Solve one problem with your students' participation.)

2.) Identify the skill students will learn.

For Example:

Today I'm going to show you how to solve problems like this without drawing pictures. (Display the written objective: “add fractions with mixed numbers without drawing.”) What are we going to learn to do today? (Point to the written objective and elicit the response, “add fractions with mixed numbers without drawing.”) That's right. We're going to learn how to add fractions with mixed numbers by drawing. We'll learn to add these kinds of problems by using numbers and symbols only. We'll begin learning to do this with “fractions place value mats” and fraction number lines.

3.) Provide rationale/meaning for the skill.

For Example:

You've learned how to add fractions with mixed numbers both using concrete materials and by drawing. These are excellent strategies for solving problems like these. However, it takes quite a bit of time to do this and you may need to be able to do solve these problems more quickly. By the time we finish, you'll be very good at solving these kinds of problems in your head using only the numbers and symbols.

**Provide Explicit Teacher Modeling**

*Purpose:* to provide students a clear teacher model of how add fractions with mixed numbers both without using concrete objects and by drawing pictures.

Learning Objective 1: Add fractions with mixed numbers using a fraction place value mats and a fraction number line.

Materials:

Teacher -

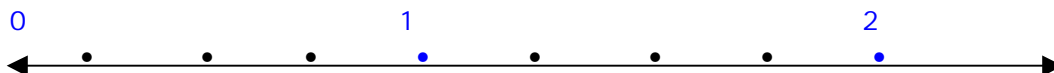
- fraction place value mat: Has the same format as a place value mat representing "ONES" and "TENS" except the column to the right is labeled "Fractions" and the column to the left is labeled "Wholes."

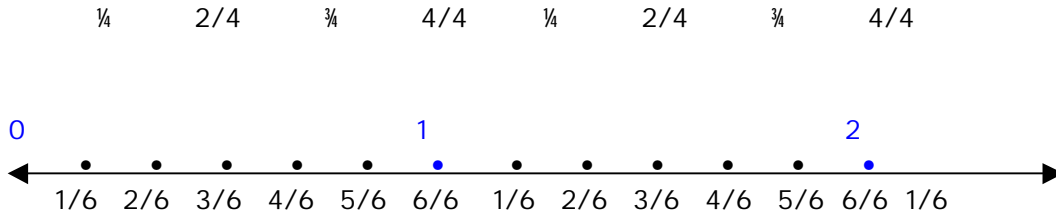
Wholes	Fractions

- "Long" fraction number lines that represent whole numbers and fractions with like denominators drawn on sentence strips:

Example of fraction number line ("fourths" and "sixths"):

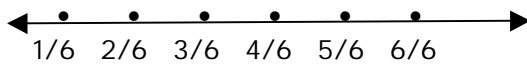
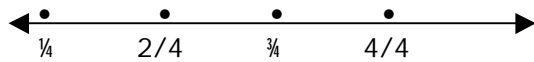
\*This example shows up to the whole number two. The actual fraction number line can extend to "10"





- “Shorter” fraction line strips that depict fractions with like denominators drawn on sentence strips (Each fraction strip should be made in the same proportion as the long fraction number line depicting whole numbers and fractions):

Example of fraction number line strips (“fourths” and “sixths”):



- A platform for visually displaying the fraction place value mats and fraction number lines so that all students can see.
- Pens/markers for writing.

*Description:*

A. Break down the skill of adding fractions with mixed numbers using a fraction place value mat and fraction number lines.

- 1.) Discover the sign/operation.
- 2.) Read the problem.
- 3.) Write the “wholes” and fractions in appropriate columns of place value mat.
- 4.) Introduce the fraction number line.
- 5.) Add fractions using fraction number line.
- 6.) Determine if one or more wholes can be made from adding the fractions by using the fraction number line.
- 7.) “Regroup” the “whole(s)” made to the “whole” column of the fraction place value mat.
- 8.) Add the wholes and write the number representing the sum.
- 9.) Say the sum/total.

B. Explicitly describe and model how to adding fractions with mixed numbers using a fraction place value mat and fraction number line.

1.) Discover the sign/operation.

For Example:

I have problem here. I know the first thing I need to do to solve an equation is to discover the sign to determine what math operation I need to use. I see I have a "plus" sign so I know I need to add. (Point to "+.") I'll circle it so I can remember I need to add.

- Think aloud
- Skim problem with finger
- Point to sign
- Circle sign

2.) Read the problem. Color-code the whole numbers and fractions according to the same color-coding used when you introduced problems at the concrete and representational levels of understanding. Color-coding can assist students with visual processing difficulties and attention problems to discriminate whole numbers from fractions when they draw. (Fading of color-coding can occur as students demonstrate understanding.)

- e.g.  $2 \frac{2}{4}$   
+  $3 \frac{3}{4}$

- Think aloud
- Point to numbers/symbols
- Prompt student thinking

For Example:

Now that I've discovered the sign and know that I need to add, the next thing I do is read the entire problem. (Read the problem aloud pointing to each whole number and fraction as you read it, then invite students to read the problem again with you.) OK, by reading the problem, I know I'm adding fractions with mixed numbers. I'm adding "two and two-fourths" plus "three and three-fourths." (Point to each mixed number as you say it.)

3.) Write the "wholes" and fractions in appropriate columns of place value mat (And introduce "fraction place value mat.")

- Introduce fraction place value mat
  - Think aloud
  - Point to characteristics of place value mat
  - Prompt student thinking - "How similar to/different from place value mat used before?"
  - Emphasize meaning of each column

For Example:

I'm going to use a special place value mat to help me add fractions with mixed numbers. You have all used place value mats before. (Show an example of a place value mat used for whole numbers with a "ones" and "tens" column. Elicit from students what the columns represent and that the place value mat was a way to learn about "regrouping" or "trading.") I'm going to use a place value mat that is very similar to this one except the place value mat I'm going to use to add fractions with mixed numbers is a "fraction place value mat." (Display the "Fraction Place Value Mat" beside the whole number place value mat.) How is the "fraction place value mat" similar to the one you have used before? (Elicit appropriate responses.) How is it different? (Elicit appropriate responses. \*Emphasize what the "fraction" column will represent and what the "wholes" column represents.)

- Write the "wholes" and fractions in appropriate columns of place value mat.
  - Think aloud
  - Point to columns as you refer to them
  - Point to numbers/fractions as you relate them to columns
  - Write numbers/symbols in appropriate columns
  - Prompt student thinking - "What operation?"

For Example:

Now that I know what the "wholes" and fractions are, I can write them on my fraction place value mat. My fraction place value mat will help me add fractions with mixed numbers without drawing pictures. To write the numbers and symbols that represent fractions, I can simply re-write the problem, putting the whole numbers in the "whole" column of my place value mat (Point to the "whole" column.) and the fractions in the "fraction" column of my place value mat. I'll do that now. (Point to the whole number in the first mixed number) This number represents two wholes, so I can write the number "two" in the "whole" column. (Write "2" in the "whole" column, and then point to the fraction in the first mixed number.) This fraction means "two-fourths." I can write "two-fourths" in the "fraction" column. (Write "2/4" in the fraction column. \*Continue this process for the remaining mixed numbers.) Now, I already discovered what operation I need to use by identifying the sign in the problem. What operation am I using? (Point to the "plus" sign and elicit the response, "addition.") Yes, I need to add these mixed numbers. I can write a plus on my place value mat to remind me that I need to add the mixed numbers I've written on my place value mat. (Write "+" in the "whole" column next to the last whole number.)

Wholes	Fractions
2	2/4

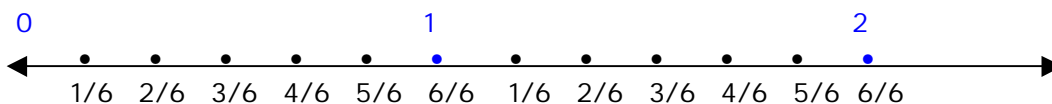
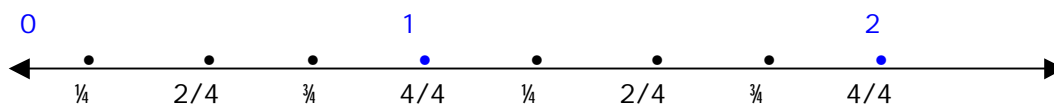
$+ 3$	$\frac{3}{4}$
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4.) Introduce the fraction number line and fraction number line strips.

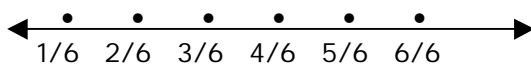
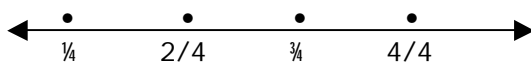
- Color-code whole numbers
- Cue features with finger

Example of fraction number line ("fourths" and "sixths"):

\*This example shows up to the whole number two. The actual fraction number line can extend to "10"



Example of fraction number line strips ("fourths" and "sixths"):



5.) Add fractions using fraction number lines.

- Think aloud
- Review drawing process and relate to abstract process
- Point to fractions/whole numbers on fraction number line as you refer to them
- Prompt student thinking - "Which fraction line do I use?"

- Represent first fraction on long number line – circle fraction
- “Add” second fraction with short number line – cue total with finger
- Write sum of fractions in appropriate column on fraction place value mat

For Example:

Now that I have my problem represented on my place value mat, I can now add. When we drew pictures, we added by combining the circles that represented wholes. We also combined the drawings of circle pieces that represented fractions to see if we could make more circles/wholes. I’m going to combine the wholes and fractions just like we did when we drew pictures, but this time I’m not going to draw pictures. I’m going to use number lines to help me add fractions with mixed numbers.

To do this, I’ll need to use the two different size number lines we just explored. Which fraction number lines should I use, halves, fourths, sixths, or eighths? (Point to each type and elicit the response, “fourths.”) That’s right, I need to use my “fourths” number lines because I am adding mixed numbers with “fourths.” (Point to the fractions on written on the place value mat, emphasizing the denominator.) Ok, when we add mixed numbers without drawing, its easiest to add the fractions first. (Point to the fractions in the “fractions” columns. This is similar to what we do when we add whole numbers that have two digits. We start by adding the ones in the “ones” column. Looking at my fractions in the “fractions” column, I see I have “two-fourths” and “three-fourths.” To add these fractions on my number line, I first need to find where the first fraction is on my long number line. To find “two-fourths” on my number line, I start at “zero” on my number line and move my finger down the number line until I find “two-fourths.” (Starting at zero, run your finger down the number line until you come to “2/4.”) I can circle this point on the number line so I know where “two-fourths” is on my number line.

I can use my short number line to add “three-fourths” to “two-fourths.” I do this by placing the beginning of my short number line directly beneath the “two-fourths” I circled on the long number line. (Place the short number line beneath the long number line so that the beginning of the short number line is directly below “2/4.”) Is my short number line positioned correctly? (Elicit the response, “yes.”) Why is it positioned correctly? (Elicit the response, “because the beginning of the short number line is directly beneath “two-fourths” on the long number line.) Great thinking! It is very important that I place the short number line so that it starts with the first fraction in my problem, in this case, “two-fourths.” To add “three-fourths” to “two-fourths,” I simply find “three-fourths” on my short number line and then see what fraction or mixed number on the long number line it is underneath. (Run your finger down the short number line to “3/4” and then point to the fraction it is under on the long number line.)

I see that when I add “three-fourths” to “two-fourths” using my number lines, I get “one and one-fourth.” (Point to the “1” at the top of the number line and then to the “1/4” that lies between “1” and “2.”) How much is “two-fourths” (Point to “2/4” on the long number line.) plus “three-fourths (Point to “3/4” on the short number line.)?” (Point to the “1” and then the “1/4” that lies between “1” and “2” on the long number line, and elicit the response,



“one and one-fourth.”) Great, by using my number lines, I know that “two-fourths” plus “three-fourths” (Point to “2/4” and “3/4” on the place value mat.) equals one whole and one-fourth. I can write “one and one-fourth” in the “fraction column” on my place value mat. (Write “1 ¼” underneath a line drawn below “3/4” in the fraction column of the place value mat.)

Wholes	Fractions
2	2/4
+ 3	<u>3/4</u>
	/ 1/4

6.) Determine if one or more wholes can be made from adding the fractions by using the fraction number line.

- Think aloud
- Point to sum in “fractions” column
- Prompt student thinking - “How many new wholes?”

For Example:

When we drew pictures and combined fractions, we added any “new” wholes with the “wholes” we already had. Do we have a “new” whole now that we added “two-fourths” and “three-fourths?” (Point to “1 ¼” and elicit the response, “Yes.”) How many new wholes do we have? (Elicit the response, “one.”) That’s correct, the “one” in “one and one-fourth” represents one whole.

7.) “Regroup” the “whole(s)” made to the “whole” column of the fraction place value mat.

- Think aloud
- Point to numbers
- Prompt student thinking
- Cross out whole number in “fraction column”

For Example:

I have to combine my new whole with my original whole numbers represented in the “wholes” column. (Point to the numbers in the “wholes” column.) Hmm, I guess I need to move the “one” over to my “wholes” column. I can do that by crossing out the “one” in “one and one-fourth” and write it in the “wholes” column. Do you agree? (Elicit the response, “yes.” \*If there is confusion, relate this process back to the drawing process again.) (Write “1” above the first number in the “wholes” column.

Wholes	Fractions
1	
2	2/4

+ 3	$\frac{3}{4}$ <del>1</del> $\frac{1}{4}$
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8.) Add the wholes and write the number representing the sum.

- Think aloud
- Write "regrouped" number in "wholes" column
- Add whole numbers and write sum

For Example:

Now all I need to do is add all my whole numbers. I can do this by adding my one "new" whole number to my original whole numbers in the "wholes" column. (Demonstrate adding the whole numbers). My total is six whole numbers. I'll draw a line under the "three" in the "wholes" column and write "six." (Draw a line underneath "3" and write "6.")

Wholes	Fractions
<del>1</del> 2	2/4
+ <u>3</u> 6	$\frac{3}{4}$ 1 1/4

9.) Say the sum/total.

- Think aloud
- Cue fractions/whole numbers with finger
- Prompt student thinking

For Example:

Let's see, I've added my fractions (Point to the "fractions" column.) and I've added my whole numbers (Point to the "wholes" column.). After doing this, my answer is "six and one-fourth." (Point to "6" and then to "1/4" as you say this.) What is the sum? (Elicit the response, "six and one-fourth." Great! "Two and two-fourths" plus "three and three-fourths" equals "six and one-fourth." (Point to each mixed number as you say it.) How many "wholes" are there? (Elicit the response, "six.") Yes, there are six wholes. (Point to the "6.") What is the fractional part? (Elicit the response, "one-fourth.") Yes, "one-fourth" is the fractional part. (Point to the "1/4.")

10.) Replicate this process at least three times for each "fractional part" (e.g. halves, fourths, eighths, etc.)

Learning Objective 2: Solve story problems and equations involving addition of fractions with mixed numbers using the FASTDRAW Strategy without drawing.

*Materials:*

Teacher -

- a visual display of the FASTDRAW and DRAW Strategy (for the "A" step in DRAW, omit "or draw and check.")

Find what you are solving for.

Ask yourself, "What is the important information?"

Set up the equation.

Iie down the sign.

Discover the sign.

Read the problem.

Answer the problem.

Write the answer.

- a visual medium for writing (e.g. chalkboard, dry-erase board, chart paper.)
- markers/pens for writing.
- prepared story problems and/or equations that represent addition of fractions with mixed numbers. Color code whole numbers and fractions consistent with color-coding used to identify these concepts in story problems/equations used at the concrete level of understanding (e.g.  $3 \frac{1}{3} + 1 \frac{2}{3} = \underline{\quad}$ .)
- fraction place value mats and fraction number lines (as needed).

A. Break down the skill of solving story problems and equations involving addition of fractions with mixed numbers using the FASTDRAW Strategy without drawing.

- 1.) Introduce story problem.
- 2.) Read the story problem aloud and then have students read it with you.

3.) Teach finding the important information in the story problem and setting up an equation using the steps "FAST" from the "FASTDRAW" Strategy.

3a. Find what you are solving for.

3b. Ask yourself, what is the important information (circle it).

3c. Set up the equation.

3d. Tie down the sign.

4.) Teach drawing solutions using the steps "DRAW" from the "FASTDRAW" strategy.

4a. Determine the sign.

4b. Read the problem.

4c. AnsWER without drawing (using fraction place value mats and fraction number lines.)

4d. Write the answer.

5.) Model how to solve the story problem by relating the "answer" to the equation back to the story problem context.

6.) Model how to draw solutions to equations by repeating the steps in #4 and #5 at least two or three more times with different division equations.

B. Explicitly describe and model how to solve story problems and equations involving addition of fractions with mixed numbers.

\*Follow the same procedures outlined for teaching and using the FASTDRAW Strategy described for the math concept "Division Process and Division with Remainders"/SOL 5.5. These procedures were also replicated at the representations/drawing level for this math concept. The primary difference will be when you model the "A" step in "DRAW." You should model using fraction place value mats and fraction number lines (See Learning Objective #1) to answer the equation instead of drawing pictures.

#### Key Ideas:

1.) Fade the use of fraction place value mats and fraction number lines to solve equations when students demonstrate mastery of adding fractions with mixed numbers using them.

2.) To fade the use of fraction number lines, teach students to "count on" using the "long" fraction number line when adding fractions without using the "short" fraction number line. When students are able to perform this to mastery, then encourage students to add fractions without the "long" number line.

3.) To fade use of the "fraction place value" mat, first fade the lines so that students simply write "F" and "W" and then re-write the equation by writing the whole numbers under "W" and the fractions under "F." When students demonstrate mastery of this, then encourage students to "regroup" wholes without naming the columns.

#### **Scaffold Instruction**

*Purpose:* to provide students the opportunity to build their initial understanding of how to add fractions with mixed numbers without concrete materials or drawings, and to provide you the opportunity to evaluate your students' level of understanding after you have initially modeled this skill.

Materials:

\*Dependent on the skill you are Scaffolding Instruction for (See the materials listed for the specific skill you want to scaffold under Explicit Teacher Modeling).

*Description:*

\*Scaffolding at the abstract level of instruction should occur using the same process as scaffolding instruction at the concrete and representational/drawing levels of instruction. (See the description of Scaffolding Instruction in the Concrete Level Instructional Plan for this math concept.) The steps listed for each skill during Explicit Teacher Modeling should be used as structure for scaffolding your instruction.

A. Scaffold instruction using a high level of teacher direction/support. Dependent on the needs of your students, you may want to continue to associate drawings to the abstract level rounding process during this phase of scaffolding. Move to the next phase of scaffolding only when students demonstrate understanding and ability to respond accurately to your prompts.

B. Scaffold instruction using a medium level of teacher direction/support. If you associated drawings with the abstract process while scaffolding using a high level of teacher direction/support, then do not include drawings during this phase of scaffolding. Move to the next phase of scaffolding only when students demonstrate understanding and ability to respond accurately to your prompts.

C. Scaffold instruction using a low level of teacher direction/support Students should actually add fractions with mixed numbers as you prompt them during this phase of Scaffolding Instruction. Move students to independent practice of the skill only after they demonstrate the ability to perform the skill with limited prompting from you.

Instructional Phase 2: Facilitate Acquisition to Mastery – Student Practice

1. Receptive/Recognition Level

*Purpose:* to provide students multiple opportunities to practice matching equations to their solution represented on place value mats.

Learning Objective1: Add fractions with mixed numbers using a “fraction place value mat” and a fraction number line.

**Self-Correcting Materials – Flash Cards**

Materials:

Teacher -

- sets of 5 x 7 note-cards with the following: Front - an equation written at the top with two or three examples of place value mats containing the numbers represented in appropriate columns and a solution. Back - a star or other cue is drawn next to the hole that represents the correct solution.

Front

$4 \frac{3}{8} + 3 \frac{6}{8} =$  **A1**

Wholes	Fractions	Wholes	Fractions	Wholes	Fractions
2		1			
4	$\frac{3}{8}$	4	$\frac{3}{8}$	4	$\frac{3}{8}$
+ 3	$\frac{6}{8}$	+ 3	$\frac{6}{8}$	+ 3	$\frac{6}{8}$
9	$\frac{2}{8}$	8	$\frac{1}{8}$	7	$\frac{9}{8}$

Back

**A1**

♦

\*To make the card development process easier, you can develop the equations and choices by writing them on sheets of copy paper to fit the size of the note-cards you are using. For example,

the above equation and choices can be made so that if cut out, they can be glued to the front side of a 5 x 7 note-card. Multiple copies of examples and choices can be made to make multiple sets of cards. Code each set of cards by representing each set with a letter and each card with the letter of the set and a number that represents the particular example/problem. Answer keys can be developed based on the letter and number for each card. You can make multiple sets of the same problems or have several sets of note-cards that represent different problems.

- answer key for each lettered set of cards.

Students -

- one set of note cards.
- sheet of paper. (Students can number the paper as they respond to the note-cards. They also should put the letter of the set at the top of their response sheet.)
- pencil

*Description:*

Activity

Students work individually with sets of 5 x 7 note cards with appropriate prompts and choices on the front side and the answer provided on the backside. Students read the equation at the top of the card and then examine the three choices underneath the equation. Each choice is a fraction place value mat with the equation represented as well as the solution. Students choose the place value mat that represents the correct solution by placing the point of their pencil in the hole next to their choice. They turn the card over to see if the hole they chose has a "star" or some other cue indicating the correct solution. Students keep track of their responses by writing their answers on a numbered sheet of notebook paper. Students also put the "letter" of the set of cards they use found at the upper right hand corner of each card. The teacher monitors students as they practice, providing specific corrective feedback, positive reinforcement, and answering questions as needed. After the activity, students turn their response sheets in and the teacher evaluates them using the answer key for each set of note-cards to check for student understanding.

Self-correcting Materials Steps:

- 1.) Introduce self-correction material.
- 2.) Distribute materials.
- 3.) Provide directions for self-correcting material, what you will do, what students will do, and reinforce any behavioral expectations for the activity.
- 4.) Provide time for students to ask questions.
- 5.) Model responding/performing skill within context of the self-correcting material.
- 6.) Model how students can keep check their responses.
- 7.) Have students practice one time so they can apply what you have modeled. Provide specific feedback/answer any additional questions as needed.
- 8.) Monitor students as they work.
- 9.) Provide ample amounts of positive reinforcement as students play.
- 10.) Provide specific corrective feedback/ re-model skill as needed.

- 11.) Encourage students to review their individual learning sheets, write the total number of "correct" responses under the "C" (Correct) column and do the same for the "H" (Help) column.
- 12.) Review individual student performance record sheets.

## 2. Expressive Level

Learning Objective 1: Add fractions with mixed numbers using a "fraction place value mat" and a fraction number line.

### **Instructional Game - Fraction Dice Game**

*Purpose*: to provide students multiple opportunities to practice adding fractions with mixed numbers.

*Materials*:

Teacher -

- pairs of dice: Each pair includes one die that has whole numbers on each side, while the other die has fractions with like denominators on each side. Blank labels can be used to make numbers and fractions. Cut out a piece of label that fits the side of a die and write the appropriate whole number or fraction on it. Glue the numbers and fractions to the sides of the dice. \*See "Alternative Ideas for Prompts" under "Description" for additional ideas.
- game board (optional)
- sets of Chance cards with different consequences written on them (e.g. add 3pts to your score; move ahead 2 spaces; subtract 2 pts from your score, move back 3 spaces.)
- 

Students -

- two pair of dice; two spinners; two sets of cards depicting whole numbers and fractions with like denominators.
- one game board per small group (optional)
- one set of Chance Cards per small group
- each student has a copy of a DRAW Strategy Cue Sheet (\*includes steps of the DRAW Strategy.)
- each student two sheets of notebook paper with one labeled "play" and one labeled "check."
- pencil

*Description*:

#### Activity

Students can play in pairs or small groups. Each small group has two pair of dice. One die has whole numbers on each side (e.g. 1, 2, 3, 4, 5, 6) and one die has fractions with like denominators (e.g.  $\frac{1}{4}$ ,  $\frac{2}{4}$ ,  $\frac{3}{4}$ ,  $\frac{4}{4}$ ). Each player rolls each pair of dice one at a time. With each roll, the player writes the mixed number represented by the pair of dice in the form of an addition equation on their "play" sheet. The player adds the two mixed numbers using a DRAW Strategy Cue Sheet as needed and writes the answer. The other players in the small group check the



player's response by solving the equation on their "check" sheet. For correct responses, each player can either receive a set number of points or move a game piece on a game board. The winner is the student who reaches a designated total of points or who reaches the finish space on the game board. \*"Chance" cards can also be included to increase motivation. Students select a "chance" card after they respond. Each "chance" card either tells the player to add bonus points to their score/move additional spaces, or it tells them to subtract points from their score or to move back a specified number of spaces. The teacher monitors student academic and social behavior as they practice, providing positive reinforcement, providing specific corrective feedback, and answering questions as needed. Students turn in both their "play" sheets and their "check" sheets at the end of the activity. The teacher reviews individual sheets to evaluate student understanding.

#### Alternative Ideas for Prompts:

\*Two spinners can also be used, where one spinner has whole numbers listed and the other has fractions with common denominators listed (\*Spinners can be made so they have multiple sets of fractions with like denominators. A circular spinner can be divided into "bands" of fourths, sixths, and eighths where the outside "band" has sections with "fourths," the middle band has sections with "sixths," and the inside "band" has sections with "eighths." Students can be assigned particular fractions with like denominators.) Another alternative is sets of cards that have written whole numbers and fractions. Students pull one card from the whole number pile and one card from the fraction pile for each mixed number.

#### Instructional Game Steps:

- 1.) Introduce game.
- 2.) Distribute materials.
- 3.) Provide directions for game, what you will do, what students will do, and reinforce any behavioral expectations for the game.
- 4.) Provide time for students to ask questions.
- 5.) Model how to respond to the prompts.
- 6.) Provide time for students to ask questions about how to respond.
- 7.) Model how students can keep track of their responses, how check other student's responses, and how to give and receive corrective feedback.
- 8.) Play one practice round so students can apply what you have modeled. Provide specific feedback/answer any additional questions as needed.
- 9.) Monitor students as they practice by circulating the room, providing ample amounts of positive reinforcement as students play, providing specific corrective feedback/ re-modeling skill as needed.
- 10.) Play game.
- 11.) Encourage students to review their individual response sheets, write the total number of "correct" responses under the "C" (Correct) column and do the same for the "H" (Help) column.
- 12.) Review individual student response sheets to determine level of understanding/proficiency and to determine whether additional modeling from you.

Instructional Phase 3: Evaluation of Student Learning/Performance (Initial Acquisition through Mastery/Maintenance)

### **1. Continuously Monitor & Chart Student Performance**

*Purpose:* to provide you with continuous data for evaluating student learning and whether your instruction is effective. It also provides students a way to visualize their learning/progress.

*Materials:*

Teacher –

- appropriate prompts if they will be oral prompts
- appropriate visual cues when prompting orally

Student –

- appropriate response sheet/curriculum slice/probe
- graph/chart

*Description:*

Steps for Conducting Continuous Monitoring and Charting of Student Performance:

- 1.) Choose whether students should be evaluated at the receptive/recognition level or the expressive level.
- 2.) Choose an appropriate criteria to indicate mastery.
- 3.) Provide appropriate number of prompts in an appropriate format (receptive/recognition or expressive) so students can respond.
  - Based on the skill, your students' learning characteristics, and your preference, the curriculum slice or probe could be written in nature (e.g. a sheet with appropriate prompts; index cards with appropriate prompts), or oral in nature with visual cues (e.g. say and point to problems on board as students respond in writing or orally to the same problems written on their individual curriculum slice.).
- 4.) Distribute to students the curriculum slice/probe/response sheet/concrete materials.
- 5.) Give directions.
- 6.) Conduct evaluation.
- 7.) Count corrects and incorrects/mistakes (you and/or students can do this depending on the type of curriculum slice/probe used – see step #3).
- 8.) You and/or students plot their scores on a suitable graph/chart. A goal line that represents the proficiency (see step # 10 for abstract level skill proficiency criteria) should be visible on each students' graph/chart.
- 9.) Discuss with children their progress as it relates to the goal line and their previous performance. Prompt them to self-evaluate.
- 10.) Evaluate whether student(s) is ready to move to the next level of understanding or has mastered the skill at the abstract level using the following guide:

Abstract Level: demonstrates near 100% accuracy for adding fractions with mixed numbers (two or fewer incorrects/mistakes) and a rate (# of corrects per minute) that will allow them to be successful when using that skill to solve real-life problems and when using the skill for higher level mathematics that require use of that skill.

11.) Determine whether you need to alter or modify your instruction based on student performance.

2. Additional Assessment Activity Appropriate For This Math Skill/Concept

\*This assessment activity can be used with students who demonstrate difficulty with adding fractions with mixed numbers at the abstract level.

*Purpose:* to assess where student understanding of adding fractions with mixed numbers is "breaking down."

### **Flexible Math Interview/C-R-A Assessment**

#### *Materials:*

Teacher –

- appropriate concrete materials for adding fractions with mixed numbers (See Concrete Level Instructional Plan – Explicit Teacher Modeling.).
- appropriate examples for assessment (equations/story problems)
- paper to record notes.

#### *Description:*

Have students solve problems using concrete materials, by drawing, and without concrete materials or drawings. Ask students to explain their answers as they respond. Note where in the problem-solving process students "break down;" both, at what level they begin having difficulty, and at what point within that level of understanding they demonstrate misunderstanding/non-understanding. Based on where students demonstrate difficulty, provide explicit teacher modeling at that level of understanding and for the particular sub-skill they are having difficulty with. As the student demonstrates understanding, scaffold your instruction until they are ready to practice the skill independently. As students demonstrate mastery of the skill at that level of understanding, then provide explicit teacher modeling at the next level of understanding. Follow this process until students demonstrate mastery at the abstract level.

#### Key Ideas

- 1.) Students who demonstrate difficulty at the abstract level of understanding may have "gaps" in their understanding that can be traced back to their representational/drawing level of understanding or even their concrete level of understanding. By providing additional teacher modeling at the level their "gap" in understanding began and then moving them from a concrete-to-representational-to-abstract level of understanding, you can assist students to become successful at the abstract level of understanding.

- 2.) Sometimes students demonstrate difficulty at the abstract level because they did not receive enough practice opportunities at the concrete and representational/drawing levels. The drawing level is a very important step for these students. Some students need continued practice drawing solutions and associating their drawings to the abstract symbols and the mental processes necessary to perform at the abstract level.
  
- 3.) Some students understand the concept, but have difficulty remembering the steps necessary to perform the skill at the abstract level. Providing students with cues they can refer to as they practice at the representational/drawing and abstract levels of instruction is very helpful (e.g. DRAW Strategy). Such cueing provides them the independence to practice. Multiple practice opportunities translate into repetition, and repetition enhances memory. The use of instructional games and self-correcting materials are an excellent way to provide students with multiple opportunities to solve division problems.
  
- 4.) Helping your students build their fluency for solving equations can also increase their abstract level problem-solving efficiency. Providing daily one-minute timings and charting student performance is an effective way to do this. It is important to both communicate with students what their "learning pictures" (charts) mean and to set short-term achievable goals. Seeing "what" they are striving for and seeing their progress as they move toward a goal is very motivating for children! (See the description of the instructional strategy "Continuous Monitoring and Charting Student Performance" for more information. This description can be found by clicking on "Instructional Strategies" on the main MathVIDS menu bar found on your left panel.)
  
- 5.) Enhancing the "meaningfulness" of abstract equations can also aid students who are having difficulty achieving mastery at the abstract level both by providing them a deeper level of conceptual understanding and by enhancing their memory of the problem-solving process. One approach you might try is to reinforce what the numbers and symbols mean using language. By modeling language (and encouraging students to use their own language) that describes what each number and symbol represents, students can gain a deeper level of understanding of the "abstract process" they are struggling to master.

For Example:

$$\begin{array}{ccccccc}
 2 & 1/4 & & + & & 3 & 3/4 & = & \underline{\quad} \\
 \text{circles} & & & \text{combined with} & & \text{circles} & & \text{totals} & 
 \end{array}$$

Provide your students multiple opportunities to use their language as they practice solving equations. As students practice, they have the opportunity to associate "meaning" to the abstract process.

## Instructional Phase 4: Maintenance – Periodic Practice to Maintain Student Mastery of Skills

\*Maintenance activities at the abstract level of understanding should include concrete and representational/drawing experiences as well as “abstract” (numbers and symbols only) experiences. By “re-visiting” previous concrete and representational/drawing experiences, students reinforce the conceptual understanding they acquired during those phases of instruction. Including “language experiences” during these maintenance activities, where students describe their solutions, also reinforces conceptual understanding students established during their concrete and representational/drawing experiences.

*Purpose:* to provide students with opportunities to maintain their level of mastery for adding fractions with mixed numbers.

### 1. Instructional Games & Self-Correcting Materials

*Materials:*

\*Dependent on the Instructional Games or Self-Correcting Materials you implement.

*Description:*

\*Periodically provide students opportunities to practice division with and without remainders via self-correcting materials and instructional games. This can be done via “centers,” in small groups, or as a whole class. Include opportunities to solve division problems with concrete materials and by drawing in addition to abstract level practice opportunities. Even though students master a concept/skill at an abstract level, providing maintenance practice opportunities using concrete materials and by drawing reinforce their conceptual understanding. (\*See the descriptions for “Instructional Games” and “Self-Correcting Materials” for more information of how to implement these student practice strategies.)

### 2. Problem of the Day

*Materials:*

Teacher –

- a written prompt on the chalkboard, dry-erase board, or overhead projector (e.g. an equation involving addition of fractions with mixed numbers or story problem) or a concrete/drawing example representing a solution to a problem involving addition of fractions with mixed numbers.

Students –

- paper and pencil to record their responses

*Description:*

Teacher presents a “problem of the day” that focuses on a particular skill or conceptual understanding of solving problems involving addition of fractions with mixed numbers. The problem can be written in nature where students solve the problem with concrete materials, by drawing, or at the abstract level only. Students can also be challenged to develop a story problem for an already solved equation. The “problem of the day” is displayed as students enter the room or as the period begins. Students are asked to “solve” the problem and provided necessary directions. After an appropriate amount of time (e.g. 2-3 minutes), the teacher and the students “talk through” the problem and its solution. Students can individually describe how they approached the problem. Specific positive verbal reinforcement is provided by the teacher as well as specific feedback regarding misunderstandings students may have. Teacher notes students who seem to be having difficulty for the purpose of reviewing/re-modeling appropriate skills and concepts.

Ideas for Prompts:

- 1.) Display the concrete or drawing representation of an equation as well as its solution and ask students to represent the equation and the solution using only numbers and symbols.
- 2.) Display an equation and ask students to represent the equation and the solution with concrete materials or drawings.
- 3.) Display a concrete, drawing, or abstract representations of an equation and have students develop a story problem for that equation.
- 4.) Display an equation and solution with concrete materials, by drawing, or with only numbers and symbols with one part of equation missing (e.g. one of the mixed numbers being added) and ask students to determine the missing part.