Instructional Plan
Concrete Level

Name of Math Skill/Concept: Adding Fractions with Mixed Numbers (Like Denominators)

Prerequisite Skills Needed:

1. Ability to identify concrete representations of fractional parts and wholes.
2. Ability to add common fractions with like denominators.

Learning Objectives:

1. Combine two sets of concrete materials that represent fractions with mixed numbers.
2. Solve story problems involving addition of fractions with mixed numbers using concrete materials.
3. Solve equations involving addition of fractions with mixed numbers using concrete materials.

Important Ideas for Implementing This Teaching Plan:

1.) It is important to first provide students with "pure" concrete experiences where students combine sets of concrete objects that represent fractions with mixed numbers (e.g. sets that include both wholes and fractional parts.) After students demonstrate mastery for combining sets with "wholes" and fractional parts, then begin explicitly teaching/relating the mathematical language and mathematical symbols in conjunction with the concrete materials.

Instructional Phase 1: Initial Acquisition of Skill/Concept – Teacher Directed Instruction

Teach Skill/Concept within Authentic Context

Description:
Determining how much pizza is left over after a class party serves as a context for introducing and providing explicit teacher modeling for addition of fractions with mixed numbers. "Pizzas" made from tag-board are used to re-create the story context that provides the problem solving situation.

Build Meaningful Student Connections

Purpose: to assist students to build meaningful connections between what they know about adding common fractions with concrete materials to adding fractions with mixed numbers.

Learning objective 1: Combine two sets of concrete materials that represent fractions with mixed numbers.

Materials:

Teacher –

- concrete materials that represent fractions (e.g. circle pieces, fraction strips, cuisenaire rods.)
- visual platform for showing concrete materials (e.g. floor where children can circle around; place magnetic strips on back of concrete materials and demonstrate them on the chalkboard or dry-erase board.)
- language card that reads: "mixed numbers." (*by including simple drawings of concrete materials that represent several examples of mixed numbers, students with reading difficulties can be provided a meaningful cue.)
- written display of learning objective: "Add fractions with mixed numbers using concrete materials." (*Highlight "mixed numbers" in the written objective.)

Description:

1.) Link to students' prior knowledge of concrete representations of fractions.

For Example:

Let's review some things about fractions using concrete materials. (Hold up a circle piece.) What is this? (Elicit the response, "a circle.") Yes, it's a circle. Does this circle represent a whole or a fractional part of a whole? (Elicit the response, "a whole.") Yes, this circle represents a whole. (Hold up a "1/2" piece and place it on top of the circle.) What does this piece represent? (Elicit the response, "one-half.") Yes, this piece represents "one-half." It takes up one-half of the space of the whole circle. (*Continue this process for several more fraction pieces – e.g. "1/4," and "1/8.")

You already know how to add fractions like "one-fourth" and "two-fourths." Let's review this using circle pieces. (Place down a "one-fourth" piece and then place down two "one-fourth" pieces.) I have "one-fourth" (Point to the "one-fourth" piece) and I want to add or combine it with "two-fourths" (Point to the two one-fourth pieces.) To combine them, I simply put them together to make as much of a circle as I can. (Place the "one-fourth" piece adjacent to the pieces representing "two-fourths" so that they represent three-fourths of a circle.) How many fourths do we have altogether? (Elicit the response, "three-fourths.") Yes, "one-fourth" plus "two-fourths" equals "three-fourths." I know that my fraction pieces equal three-fourths because they make up three-fourths of a whole circle. (Place the three "one-fourth" pieces on top of a whole circle to show/review this relationship.)

2.) Identify the skill students will learn: Addition of fractions with mixed numbers using concrete materials.

For Example:

We know how to add two fractions that represent parts of a whole, like "one-fourth plus two-fourths." Today we are going to learn how to add or combine concrete materials that represent both wholes and fractional parts of wholes. Let me show you what I mean. (Show two groups of concrete materials where each group has whole pieces and a fractional part – e.g. two circles and a one-fourth piece.) When we have groups like these that have both wholes and fractional parts, we call them "mixed numbers" (Point to a written display that reads, "mixed numbers.") When we add concrete materials that represent mixed numbers, we add fractions with mixed numbers.

(Point to a written display of the learning objective, "add fractions with mixed numbers using concrete materials.") What are we going to learn to do today? (Point to the learning objective and elicit the response, "addition of fractions with mixed numbers."). That's right, we are going to learn how to add fractions with mixed numbers.

3.) P rovide rationale/meaning for learning the skill.

For Example:
Being able to add fractions helps us in a lot of situations. You already know situations where adding fractions can come in handy, like when you and some friends buy….. Being able to add fractions with mixed numbers is also a helpful thing because sometimes you have situations where you have both wholes and fractional parts you need to add. For example, say we have a pizza party in class as a celebration for our hard work learning about adding fractions with mixed numbers. After the party, we will probably have some pizza left over. We may have several slices of pizza left over as well as several whole pizzas left over. Being able to add them to see how much pizza we have total would allow us to determine whether there was enough pizza left over to give to another class.

Provide Explicit Teacher Modeling

Purpose: to provide students a clear teacher model of how to add fractions with mixed numbers using concrete materials.

Learning Objective 1: Combine two sets of concrete materials (circle pieces) representing mixed numbers with like denominators.

Materials:
Teacher –

- concrete materials (area model) to represent fractions (e.g. circle pieces).
- a platform for displaying concrete materials so all students can see (e.g. floor; chalkboard/dry-erase board with concrete materials that have adhesive magnetic strips attached to their backs).
- prepare "sets" of concrete objects that represent fractions with mixed numbers that will be combined.

Description:

A. Break down the skill of combining two sets of concrete materials representing mixed numbers.

1.) Identify the fractional parts.
2.) Combine the "wholes."
3.) Combine the fractional parts into "wholes."
4.) Add the "wholes."
5.) Add the fractional part that remains.
6.) Say what the total/sum means.

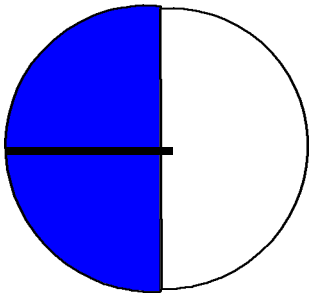
B. Explicitly describe and model how to combine two sets of concrete materials representing mixed numbers. (e.g. a set with one two whole circles and two one-fourth pieces and a set with one whole circle and three one-fourth pieces.)

1.) Model how to identify the fractional parts.

- Think aloud
- Cue relationship of fraction pieces to whole
- Prompt student thinking

For Example:

I have two groups of circle pieces here. The first thing I need to do is identify the fraction pieces in each group. Let's see, in the first group I have two whole circles and two circle pieces. I know the whole circles represent two "wholes." How can I find out what the circle pieces represent? Oh, I know how to do this. I remember that when I learned about fractions, I found I could determine what part of a whole circle a circle piece was by placing it on top of a whole circle piece. I'll take my two circle pieces and do that now. (Model placing the two one-fourth pieces on top of the whole circle piece on at a time.)



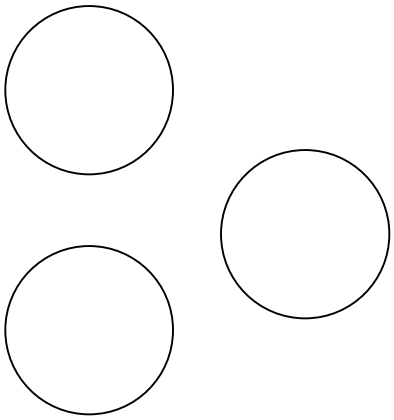
I see that two of my pieces equal half of my whole circle. If I had two more pieces like these pieces then I could cover the whole circle. If four equal parts equal a whole, what do we call each part? (Elicit the response, "one-fourth.") That's right. That means my two circle pieces in this group are one-fourth pieces. (*Follow this same process for the second group of circles and circle pieces.)

2.) Model how to combine the "wholes."

- Think aloud
- Point as you count aloud
- Prompt student thinking –"How many?"

For Example:

Now that I have identified my circle pieces, I need to combine them since we are adding these circle pieces. An easy way to do this is to put similar pieces together. Well, I have three whole circles and five one-fourth pieces. I can combine my whole circle pieces first. I'll do that now. (Move the three circle pieces in one group.) I can now count them. (Count the whole circle pieces aloud.) Ok, I have three whole circle pieces. How many whole circle pieces do I have? (Elicit the response, "three.") That's correct, I have three whole circle pieces. (Count them aloud one more time, pointing to each one as you count.)

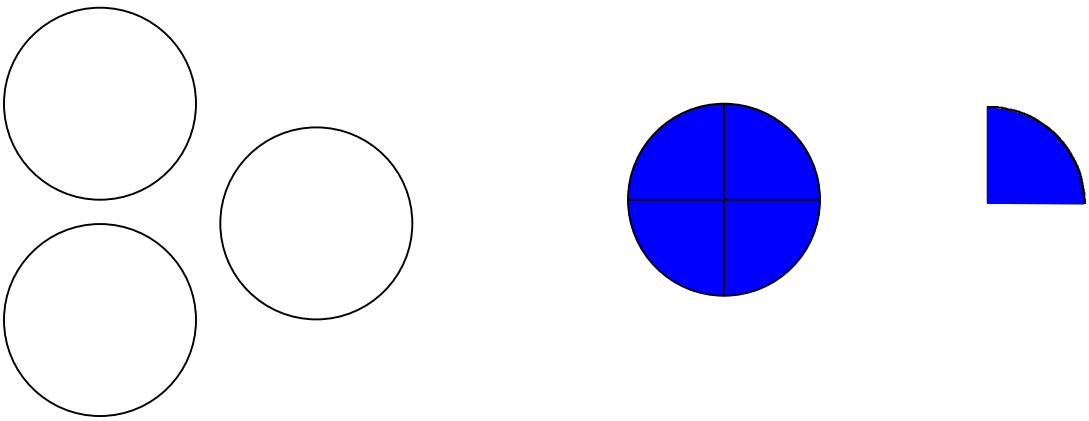


3.) Model how to combine the fractional parts into "wholes."

- Think aloud
- Place fraction pieces side to side
- Cue pieces make a whole with finger

For Example:

Now, I need to combine my circle pieces that represent one-fourth. I can do that by placing them one by one, making as many whole circles as I can. I'll do this now. (Place the one-fourth pieces one-by-one in appropriate positions to make a whole circle. You should have one 'one-fourth'-piece remaining.)



4.) Model how to add the "wholes."

- Think aloud

- Cue by pointing with finger
- Prompt student thinking – "How many?"

For Example:
Now that I have combined the whole circle pieces together and the one-fourth pieces together, I need to check to see if I created any more whole circles with my one-fourth pieces. Well, I did make another whole circle with four of my one-fourth pieces. (Point to the four one-fourth pieces that make the whole circle, counting them aloud. Move your finger around the four pieces to show they make a whole circle.) I can combine this "additional" whole circle with the three original whole circles by moving it over here. Now I have four whole circles altogether. How many whole circles are there now? (Elicit the response, "four.") That's right, I have four whole circles now. Where did my fourth whole circle come from? (Elicit the response, "from the four one-fourth pieces you put together.") Excellent thinking! I made this fourth circle (Point to the circle.) by combining four of the one-fourth pieces I started with.

5.) Model how to add the fractional part that remains.

For Example:
Ok. We have four whole circles. How many one-fourth pieces do we have remaining? (Elicit the response, "one.") That's right, we have one one-fourth-piece remaining. (Point to the remaining one-fourth piece.)

6.) Model saying what the total/sum means.

For Example:
When we have finished combining our circle pieces, then we can say what they represent. I have four whole circles and one one-fourth piece. (Point to the circles and count them aloud and then point to the one-fourth piece and count it aloud.) This represents the total after I combined my original circle pieces. What is the total after I combined the circle pieces? (Elicit the response, "four whole circles and one one-fourth piece.) Correct. I have a sum of four whole circles and one one-fourth piece. Another way to say this is I have "four and one-fourth."

7.) Repeat this process at least three more times with different circle piece combinations.

8.) Repeat steps 1-7 with several different concrete materials that represent fractional parts (e.g. cuisenaire rods, fraction strips).

Learning Objective 2: Solve story problems involving addition of mixed numbers.

Materials:

Teacher –
- concrete materials that represent fractional parts (i.e. circles and circle pieces).

- a platform for demonstrating the use of concrete materials that is clearly visible to all students.
- prepared story problems/contexts representing addition of fractions with mixed numbers.

A. Break down the skill of solving story problems involving addition of mixed numbers into teachable/learnable steps.

1.) Read the story problem/context.
2.) Identify what is to be solved for.
3.) Identify the important information.
4.) Represent the mixed numbers with concrete objects.
5.) Combine the "wholes."
6.) Combine the fractional parts into "wholes."
7.) Add the "wholes."
8.) Add the fractional part remaining.
9.) Answer the story problem.

B. Explicitly describe and model how to solve story problems involving addition of mixed numbers.

1.) Introduce/Read the story problem/context.
- Color-code phrases that reflect the denominator and numerators
- Read aloud and point to words
- Have students read with you a second time

For Example:
The class had ordered several pizzas for a class pizza. Each pizza had eight equal slices. There is one whole pepperoni pizza left and three extra pepperoni slices. There is also one cheese pizza left and six extra cheese slices left. How many whole pizzas can be made by combining the whole pizzas and the "extra" slices?

2.) Model how to identify what is being solved for.
- Think aloud
- Skim story problem with finger
- Point to and circle question mark
- Underline question

For Example:
When I have a story problem I know the first thing I need to do after I've read it is to find what I need to solve for. Hmm, how do I do this? Oh yeah, I remember one thing that can help me find what I need to solve for. I can look for a question mark. Question marks indicate there is a question. Questions usually tell me what I need to solve for. Let me see, where is there a question mark. Aha, here is the question mark. (Point to the question mark.) When I find it, I can circle it to help me remember where it is. (Circle the question mark.) I now need to read the question, because this will tell me what I am solving for. I'll do that now. (Read the

question.) I can underline the question. That will also help me remember what I am solving for. (Underline the sentence that is the question.) What am I solving for? (Point to the question and elicit the response, "how many whole pizzas can be made by combining the whole pizza and the extra slices?") Yes, I need to find out how many whole pizzas can be made by combining the whole pizza and the extra slices.

3.) Model how to identify the important information.

- Think aloud
- Skim story problem with finger
- Point to and circle important information
- Repeat what needs to be solve for

For Example:
Now that I have found what I am solving for, I now need to read back through the story problem to find the important information. One kind of important information is a number phrase. I can find the number phrases by simply reading each sentence and asking myself is there a number phrase? I'll do that now. (Read the first sentence aloud and ask yourself, "Is there a number phrase?") Well, yes there is. It says here that there are eight equal slices in each pizza. I'll circle the number phrase, "eight equal slices." So, this number phrase tells me that each pizza is separated into eight equal parts or slices. (Read each remaining sentence and ask aloud the question, "Is there a number phrase?" When there is a number phrase, identify it and then circle it. Also "think aloud" what it represents.) Now that I have found all of the number phrases, I need to look back at the question to see what it is I need to solve. (Point to the underlined question and read it aloud.) It says I need to find out how many whole pizzas and extra slices there all if I combined them.

4.) Model how to represent the mixed numbers with concrete objects.

- introduce concrete materials

For Example:
To answer this question, I can use these "pizzas" and "pizza slices" made out of construction paper or tag-board. (If these items are not available then using circles and circle pieces to represent the pizza is also appropriate.)

- model representing each circled number phrase

For Example:
In order to find the total number of pizzas and extra pizza slices there are if we combine them all, I need to represent each of the pizzas and pizza slices I circled in the story problem. I can do this by reading each number phrase that represents whole pizzas or pizza slices; then I can use my pizzas and pizza slice to represent each number phrase. When I represent the pizza and pizza slices, I will first put them in groups based on type of pizza. This is similar to what you have done already when you practiced combining sets of whole circles and circle pieces. (Model reading each circled "number phrase" that represents the whole pizzas and pizza slices. *Be

sure to "think aloud" why the first number phrase, "eight equal slices" does not need to be represented.)

5.) Model how to combine the "wholes."

- Think aloud
- Cue with finger and word cards
- Prompt student thinking

For Example:
Now that I have my whole pizzas and pizza slices represented according to whether they are pepperoni or cheese, I need to combine all the whole pizzas I have. Well, I have one whole pepperoni pizza and one whole cheese pizza so I can put them together in a group. (Point to each whole pizza and then move them together to form a group of whole pizzas. (*Placing them next to a card that reads "whole" may be a helpful cue for some students.) How many whole pizzas do I have? (Elicit the response, "two.") Yes, I have two whole pizzas. I have one pepperoni pizza and one cheese pizza. (Point to each pizza as you say this and then count aloud, "one, two.")

6.) Model how to combine the fractional parts into "wholes."

- Prompt students to link their prior experiences of combining concrete objects that represent wholes and fractional parts to this story problem.

For Example:
Now that I have combined my whole pizzas, now I need to combine my pizza slices. This is just like we learned to do when we combined our whole circles and our circle pieces. Who remembers the particular way that we did this? (Elicit the response, "we placed them side-by-side to make a circle.") That's right, we placed our circle pieces side-by-side to make a circle.

- Prompt students to relate what fractional part each slice of pizza represents.

For Example:
How many pizza slices will it take to make a whole pizza? (Elicit the response, "eight.") That's correct, but how do you know this? (Elicit the response, "because the number phrase "eight equal slices" says how many slices are in each pizza.") Excellent thinking! The first sentence in our story problem says there are eight equal slices in each pizza. If there are eight equal slices in each whole pizza, then what fractional part is one slice of pizza? (Elicit the response, "one-eighth.") That's right, each slice of pizza is "one-eighth" of a whole pizza.

- Model combining slices to make whole pizzas.

For Example:
To combine the slices of pizza, I will place them side-by-side to form a whole pizza. Because I know each slice represents one-eighth of a whole pizza, I know it will take eight slices to make a whole pizza. How many "one-eighth" slices will it take to make a whole pizza? (Elicit the

response, "eight.") Yes, it will take eight slices. Let's see, I have three pepperoni slices and I have six cheese slices. I'll take the three pepperoni slices and place them side-by-side. (Position the three "pepperoni slices" side-by-side so they form "three-eighths" of a whole pizza.) Now I'll start adding the six cheese slices. (Position the "cheese slices" one-by-one until a whole pizza is made. One slice of "cheese pizza" should be left over. By combining the three pepperoni slices and the six cheese slices, I have one whole pizza and one extra slice of pizza. (Point to the newly formed whole pizza and then point to the "extra slice.") How many whole pizzas and extra slices did combining the three pepperoni slices and the six cheese slices make? (Elicit the response, "one whole pizza and one extra slice.") That's correct. I have made one whole pizza and I have one extra slice of pizza left over. (Point to the whole pizza and extra slice again.)

7.) Model how to add the "wholes."

- Think aloud
- Cue with finger
- Prompt student thinking – "How many?"

For Example:

When you were practicing combining circles and circle pieces, you learned that after you combined the circle pieces to make whole circles, then you added all of the whole circles together. I need to do the same thing with my pizzas. How many whole pizzas did I have to begin with. (Point to the original whole pizzas and elicit the response, "two.") That's right, I had two whole pizzas to begin with. Now I have one more. (Point to the newly formed whole pizza.) How many whole pizzas are there now? (Elicit the response, "three.") Yes, I have three whole pizzas. (Point to each whole pizza and count them aloud.)

8.) Model how to add the fractional part that remains.

- Think aloud
- Cue with finger
- Prompt student thinking – "How much left?"

For Example:

I also have an extra slice left. I need to count this extra slice as well. What fractional part of a whole pizza does this slice represent? (Elicit the response, "one-eighth.") That's correct. I have one one-eighth piece of pizza left.

9.) Model how to answer the story problem.

- Think aloud
- Count aloud with students
- Cue with finger
- Prompt student thinking
- Write solution in word form

For Example:

Now that I have combined or added all the pizza wholes and pizza slices together, I can answer the question asked by the story problem. What is the question we need to solve for? (Point to

the question in the story problem and elicit the response, "how many whole pizzas can be made by combining the whole pizzas and the "extra" slices?") Good. To answer this question, all I need to do is count the total number of whole pizzas I made and decide what fractional part of a whole pizza any extra pieces represent. Let's count the whole pizzas again. (Count aloud the whole pizzas with your students.) We have three whole pizzas. How many pizza slices do we have left? (Elicit the response, "one.") Yes, we have one slice left over. What fractional part of a whole pizza does this one slice represent? (Elicit the response, "one-eighth.") That's correct, the one extra slice of pizza represents "one-eighth." What is the question we need to answer again? (Elicit the response, "how many whole pizzas can be make by combining the whole pizzas and the "extra" slices?") Great! The answer to this question is three and one-eighth pizzas. (Write the phrase, "three and one-eighth pizzas" on the chalkboard/dry-erase board.) "Three" represents the three whole pizzas. (Point to the "three" in the written phrase and then point to the three whole pizzas.) The "one-eighth" represents the extra slice. (Point to the "one-eighth" in the written phrase and then point to the "extra pizza slice.") What does the "three" in my answer represent? (Point to the "three" in the written phrase and elicit the response, "the three whole pizzas.") Yes, the "three" in my answer means the three whole pizzas. What does the "one-eighth" in my answer represent? (Elicit the response, "the left over slice of pizza.") Excellent!

10.) Repeat steps 1-9 at least three more times with different examples using circles and circle pieces.

11.) Repeat steps 1-9 with at least one more type of concrete material that represents wholes and fractional parts.

Learning Objective 3: Solve equations involving addition of fractions with mixed numbers using concrete objects.

A. Break down the skill of solving equations involving addition of fractions with mixed numbers using concrete materials into teachable/learnable parts.

1.) Discover the sign/operation.
2.) Read the equation and identify the wholes and fractional parts.
3.) Represent the wholes and fractional parts for each mixed number with concrete materials
4.) Combine the whole pieces.
5.) Combine the fractional parts/pieces to make as many wholes as possible.
6.) Combine "new wholes" made with original whole pieces.
7.) Combine the remaining fractional pieces.
8.) Say what the total/sum means and write the solution.

B. Explicitly Describe and Model how to solve equations involving addition of fractions with mixed numbers – e.g. 4 2/4

\+ 2 3/4

1.) Discover the sign/operation.

- Think aloud
- Cue with finger
- Circle sign
- Say what sign means

For Example:

When I have an equation to solve, the first thing I need to do is to decide whether I am adding, subtracting, multiplying, or dividing. I can decide this by finding the operation sign. Hmm, I wonder where the operation sign in this equation is (run your finger along the length of the equation). Oh, here it is. (Point to the "+" sign.) This sign tells me to add, so I know I need to add these mixed numbers. I'll circle the "plus/addition" sign to help me remember I need to add.

2.) Read the problem and identify the wholes and fractional parts represented by the problem. – Color code the whole numbers and fractions according to the same color-coding used when you introduced story problems. Color-coding can assist students with visual processing difficulties and attention problems to discriminate whole numbers from fractions when they draw.

- Think aloud
- Point to number/symbols as you read
- Prompt student thinking – what numbers represent

For Example:

When I've discovered the operation I need to use, the next thing I need to do is read the problem. Reading the problem will help me determine what whole numbers and fractions I am working with. I'll read the problem aloud first and then I want you to read it with me a second time. (Read the problem aloud, pointing to each whole number and fraction as you read, then repeat the same process a second time when reading aloud with your students.) By reading the problem, I know I am adding, or combining, whole numbers with fractions. (Point to each whole number and fraction.)

Now that I've read the problem and know that I am adding whole numbers with fractions, I need to identify the wholes and the fractional parts the written numbers and fractions represent. Hmm, what is it that I am combining or adding? Well my first group is "four and two-fourths." The number "four" represents four wholes and the fraction "two-fourths" represents two-fourths of a whole. Another way to say this is that "two-fourths" equals two out of four equal parts of one whole. What does the number "four" represent? (Point to the "4" and elicit the response, "four wholes.") Yes, the number "four" equals four wholes. What does the fraction "two-fourths" represent? (Point to "2/4" and elicit the response, "two-fourths of a whole," or "two of four equal parts.") Great thinking! The fraction "two-fourths" means two-fourths of a whole or two of four equal parts. (Repeat this same process for the second "group"/mixed number set - 2 ¾.)

3.) For steps 3-8, follow the same process described for using concrete materials to solve story problems involving addition of fractions with mixed numbers (See Learning Objective 2, steps 4 – 11.). *The only difference is that you will model writing the numerical solution to the equation after you have solved it using concrete materials rather than saying the solution to a story problem.

Scaffold Instruction

Purpose: to provide students the opportunity to build their initial understanding of the division process, with and without remainders, and to provide you the opportunity to evaluate your students' level of understanding after your initial modeling of these skills.

Learning Objective 1: Combine two sets of concrete materials that represent fractions with mixed numbers.

Materials:

Teacher –

- concrete materials (area model) to represent fractions (e.g. circle pieces).
- a platform for displaying concrete materials so all students can see (e.g. floor; chalkboard/dry-erase board with concrete materials that have adhesive magnetic strips attached to their backs).
- prepare "sets" of concrete objects that represent fractions with mixed numbers that will be combined.

Description:

1.) Scaffold Using a High Level of Teacher Direction/Support

a. Choose one or two places in the problem-solving sequence to invite student responses. Have these choices in mind before you begin scaffolding instruction. (Examples of choices are shown in red.)

- Model how to identify the fractional parts.

  - "I have two groups of circle pieces here. The first thing I need to do is identify the fraction pieces in each group. Let's see, in the first group I have two whole circles and two circle pieces. What does the whole circles represent? (Elicit the response, "two wholes.") Yes, the two circles represent two "wholes." How can I find out what the circle pieces represent? Oh, I know how to do this. I remember that when I learned about fractions, I found I could determine what part of a whole circle a circle piece was by placing it on top of a whole circle piece. I'll take my two circle pieces and do that

now. (Model placing the two one-fourth pieces on top of the whole circle piece on at a time.)

What do the two fraction pieces represent together? (Elicit the response, "one-half.") Yes, two of my pieces equals half of my whole circle. If I had two more pieces like these pieces then I could cover the whole circle. If four equal parts equal a whole, what do we call each part? (Elicit the response, "one-fourth.") That's right. That means my two circle pieces in this group are one-fourth pieces. (*Follow this same process for the second group of circles and circle pieces. Elicit student responses at the same points as for the first set of concrete objects.)"

- Model how to combine the "wholes."
    - "Now that I have identified my circle pieces, I need to combine them since we are adding these circle pieces. An easy way to do this is to put similar pieces together. Well, I have three whole circles and five one-fourth pieces. Which pieces should I combine first, the whole pieces or the pieces that represent fractional parts. (Elicit the response, "the whole pieces.") Good thinking, I can combine my whole circle pieces first. I'll do that now. (Move the three circle pieces in one group.) Count them aloud with me. (Point to each circle as you count them aloud with your students.) Ok, I have three whole circle pieces. How many whole circle pieces do I have? (Elicit the response, "three.") That's correct, I have three whole circle pieces."

- Model how to combine the fractional parts into "wholes."
    - "Now, I need to combine my circle pieces that represent 'one-fourth'. I can do that by placing them one by one, making as many whole circles as I can. I'll do this now. (Place the one-fourth pieces one-by-one in appropriate positions to make a whole circle. You should have one 'one-fourth' piece remaining.)"

- Model how to add the "wholes."
    - "Now that I've combined the whole circle pieces together and the one-fourth pieces together, I need to check to see if I created any more whole circles with my one-fourth pieces. Did I make another whole circle with four of my one-fourth pieces? (Point to the four one-fourth pieces that make the whole circle counting them aloud, and elicit the response, "yes.") Yes, I did make another 'whole' with my one-fourth pieces. (Move your finger around the four pieces to show they make a whole circle.) I can combine this "additional" whole circle with the three original whole circles by moving it over here. Now I have four whole circles altogether. How many whole circles are there now? (Elicit the response, "four.") That's right, I have four whole circles now. Where did my fourth whole circle come from? (Elicit the response, "from the four one-fourth pieces you put together.") Excellent thinking! I made this fourth circle (Point to the circle.) by combining four of the one-fourth pieces I started with."

- Model how to add the fractional part that remains.

    - "Ok. We have four whole circles. How many one-fourth pieces do we have remaining? (Point to the remaining one-fourth piece and elicit the response, "one.") That's right, we have one 'one-fourth' piece remaining."

- Model saying what the total/sum means.

    - "When we have finished combining our circle pieces, then we can say what they represent. I have four whole circles and one one-fourth piece. (Point to the circles and count them aloud and then point to the one-fourth piece and count it aloud.) This represents the total after I combined my original circle pieces. What is the total after I combined the circle pieces? (Elicit the response, "four whole circles and one one-fourth piece.) Correct. I have a sum of four whole circles and one one-fourth piece."

b. Maintain a high level of teacher direction/support for another example if students demonstrate misunderstanding/non-understanding; move to a medium level of teacher direction/support if students respond appropriately to the selected questions/prompts.

2.) Scaffold Using a Medium Level of Teacher Direction/Support

a. Choose several more places in the problem-solving sequence to invite student responses. Have these choices in mind before you begin scaffolding instruction. (Examples of choices are shown in red.)

- Model how to identify the fractional parts.

    - "I have two groups of circle pieces here. What do I need to do first if I am going to add or combine the groups? (Elicit the response, "identify the fraction pieces in each group.") Excellent thinking! Let's see, in the first group I have two whole circles and two circle pieces. What does the whole circles represent? (Elicit the response, "two wholes.") Yes, the two circles represent two "wholes." How can I find out what the circle pieces represent? (Elicit the response, "place it on top of a whole circle piece." I'll take my two circle pieces and do that now. (Model placing the two one-fourth pieces on top of the whole circle piece on at a time.)

      What do the two fraction pieces represent together? (Elicit the response, "one-half.") Yes, two of my pieces equal half of my whole circle. If I had two more pieces like these pieces then I could cover the whole circle. If four equal parts equal a whole, what do we call each part? (Elicit the response, "one-fourth.") That's right. That means my two circle pieces in this group are one-fourth pieces. (*Follow this same process for the second group of circles and circle pieces. Elicit student responses at the same points as for the first set of concrete objects.)"

- Model how to combine the "wholes."

    - "Now that I have identified my circle pieces, what do I do? (Elicit the response, "combine them.") Yes, I need to combine them since we are adding these circle pieces. Which pieces should I combine first, the whole pieces or the pieces that represent fractional parts. (Elicit the response, "the whole pieces.") Good thinking. I can combine my whole circle pieces first. I'll do that now. (Move the three circle pieces in one group.) Count them aloud with me. (Point to each circle as you count them aloud with your students.) How many whole circle pieces do I have? (Elicit the response, "three.") That's correct, I have three whole circle pieces."

- Model how to combine the fractional parts into "wholes."

    - "What do I add or combine now? (Elicit the response, "the one-fourth pieces.") Great. How do I place the pieces when I combine them? (Elicit the response, place them one by one, making as many whole circles as you can.) That's right. I'll do this now. (Place the one-fourth pieces one-by-one in appropriate positions to make a whole circle. You should have one 'one-fourth' piece remaining.)"

- Model how to add the "wholes."

    - "Now that I've combined the whole circle pieces together and the one-fourth pieces together, what do I need to check for? (Elicit the response, "to see if you made any more whole circles with your one-fourth pieces.") Excellent! Did I make another whole circle with four of my one-fourth pieces? (Point to the four one-fourth pieces that make the whole circle counting them aloud, and elicit the response, "yes.") Yes, I did make another 'whole' with my one-fourth pieces. (Move your finger around the four pieces to show they make a whole circle.) How can I combine my newly made 'whole' with the other whole circles? (Elicit the response, move it with the other circles.") Good. (Move the 'new' circle.) How many whole circles are there now? (Elicit the response, "four.") That's right, I have four whole circles now. Where did my fourth whole circle come from? (Elicit the response, "from the four one-fourth pieces you put together.") Excellent thinking! I made this fourth circle (Point to the circle.) by combining four of the one-fourth pieces I started with."

- Model how to add the fractional part that remains.

    - "How many one-fourth pieces do we have remaining? (Point to the remaining one-fourth piece and elicit the response, "one.") That's right, we have one 'one-fourth' piece remaining."

- Model saying what the total/sum means.

    - "When we've finished adding/combining our circle pieces, what do we do? (Elicit the response, "say what they represent.") Yes. What is our total? (Elicit the response,

"four whole circles and one one-fourth piece.") Correct. I have a sum of four whole circles and one one-fourth piece. Another way to say that is to say 'four and one-fourth' (Point to the four circles and then the one-fourth piece as you say this.) What is another way to say four whole circles and one 'one-fourth' piece? (Elicit the response, "four and one-fourth.") Yes, we can say four and one-fourth."

b. Maintain a medium level of teacher direction/support for another example if students demonstrate misunderstanding/non-understanding; move to a low level of teacher direction/support if students respond appropriately to the selected questions/prompts.

3.) Scaffold Using a Low Level of Teacher Direction/Support

a. When students demonstrate increased competence, do not model the process. Ask students questions and encourage them to provide all responses. Direct students to replicate the process with concrete materials at their desks as you work together.

- Model how to identify the fractional parts.
    - "What do you need to do first (Elicit the response, "identify the fraction pieces in each group.") Excellent thinking! What's in the first group? (Elicit the response, "two whole circles and two circle pieces.") Good. Hold up the two circles. (Check to see all students hold up the appropriate pieces.) Great. Hold up the circle pieces. (Check to see all students hold up the appropriate pieces.) What does the whole circles represent? (Elicit the response, "two wholes.") Yes, the two circles represent two "wholes." Show me how you can I find out what the circle pieces represent."

What do the two fraction pieces represent together? (Elicit the response, "one-half.") Yes, two of my pieces equal half of my whole circle. If four equal parts equal a whole, what do we call each part? (Elicit the response, "one-fourth.") That's right. (*Follow this same process for the second group of circles and circle pieces. Elicit student responses at the same points as for the first set of concrete objects.)"

- Model how to combine the "wholes."
    - o "Now that you have identified your circle pieces, what do you do? (Elicit the response, "combine them.") Yes. Which pieces should I combine first, the whole pieces or the pieces that represent fractional parts. (Elicit the response, "the whole pieces.") Good thinking. Everybody do that now. (Check to see all students move the

circles into one group. How many whole circle pieces do I have? (Elicit the response, "three.") Good, you have three whole circle pieces."

- Model how to combine the fractional parts into "wholes."
    - "What do you add or combine now? (Elicit the response, "the one-fourth pieces.") Great. How do I place the pieces when I combine them? (Elicit the response, place them one by one, making as many whole circles as you can.) That's right. Do this now. (Check to see all students respond appropriately.)"
- Model how to add the "wholes."
    - "Now that you've combined the whole circle pieces together and the one-fourth pieces together, what do you need to check for? (Elicit the response, "to see if we made any more whole circles with our one-fourth pieces.") Excellent! Did you make any more whole circles? (Elicit the response, "yes.") How many wholes did you make? (Elicit the response, "one.") Good. How do you combine your newly made 'whole' with the other whole circles? (Elicit the response, move it with the other circles.") Good. Everybody do that now. (Check to see all students respond appropriately.) How many whole circles are there now? (Elicit the response, "four.") Fantastic!"
- Model how to add the fractional part that remains.
    - "How many one-fourth pieces do we have remaining? (Elicit the response, "one.") That's right, you have one 'one-fourth'-piece remaining. Everybody hold up the remaining one-fourth piece. (Check to see all students respond appropriately.)"
- Model saying what the total/sum means.
    - What is your total? (Elicit the response, "four whole circles and one one-fourth piece.") Correct. What is another way to say four whole circles and one 'one-fourth' piece? (Elicit the response, "four and one-fourth.") Yes, we can say four and one-fourth."

b. When you are confident students understand, ask individual students to direct the problem solving process or have the class direct you: Students ask questions and you and the students respond/perform the skill.

Instructional Phase 2: Facilitate Acquisition to Mastery – Student Practice

1. Receptive/Recognition Level

Purpose: to provide students multiple opportunities to practice selecting equations involving addition of fractions with mixed numbers that match concrete representations.

Learning Objective 3: Solving equations involving addition of fractions with mixed numbers using concrete materials.

Structured Peer Tutoring

Materials:

Teacher –

- appropriate number of concrete materials (area model) for each example.
- a set of three cards for each concrete example. Each card in the set has written on it a different equation involving addition of fractions with mixed numbers (one of which is the correct solution for the concrete example.) The equation that matches the concrete example has a star or some other cue on the back to designate it as the correct solution.
- master copy of answer key for examples
- timer of clock for keeping time.

Students –

- each student in each pair has a sheet of notebook paper to write their responses and a sheet of notebook paper to record their partner's points.
- pencils for writing

Description:

Activity

Students work in pairs. One student is the "coach" for half of the period and one student is the "player." Students switch roles for the second half of the period. Each student has a response sheet where they write their responses. Each student pair moves to each numbered example of concrete objects. The player examines the concrete objects and selects the card with the appropriate equation written on it, writing the equation they choose on their response sheet next to the appropriate number. The coach checks the player's response by checking the back of the card to see which one has a star written. The coach awards two points for correct responses. If the player responds incorrectly, the coach provides appropriate feedback. Tallies can be written on a separate sheet of notebook paper or in the margin beside each prompt. The teacher signals student pairs when it is time for students to switch roles. As students practice, the teacher circulates the room monitoring student behavior, providing specific corrective feedback, providing positive reinforcement, and prompting student thinking. Students total their points and turn in both their individual learning sheets and point sheets. The teacher reviews individual student learning sheets and point sheets to evaluate student understanding.

Structured Peer Tutoring Steps:

1.) Select pair groups and assign each pair a place to practice (try to match students of varying achievement levels if possible).
2.) Review directions for completing structured peer tutoring activity and relevant classroom rules. Practice specific peer tutoring procedures as needed (see step #4).
3.) Model how to perform the skill(s) within the context of the activity before students begin the activity. Model both what the coach does (e.g. checks answers by turning cards over; provide corrective feedback; record points) and how the player responds (e.g. examining the concrete example; choosing the card with the matching equation; writing the equation on their response sheet).
4.) Divide the practice period into two equal segments of time. One student in each pair will be the player and will respond to the examples with concrete materials. The other student will be the coach. For accurate responses, the coach awards 2 pts. For inaccurate responses, the coach provides appropriate corrective feedback
5.) Provide time for student questions.
6.) Signal students to begin.
7.) Signal students when it is time to switch roles.
8.) Monitor students as they work in pairs. Provide positive reinforcement for both "trying hard," responding appropriately, and for students using appropriate tutoring behaviors. Also provide corrective feedback and modeling as needed.
9.) Students turn in response sheets and point totals.
10.) Teacher reviews response sheets/point totals to evaluate student understanding.

2. Expressive Level

Purpose: to provide students multiple opportunities to practice combining sets of concrete materials that represent fractions with mixed numbers (Structured Peer Tutoring) or to solve equations involving addition of fractions with mixed numbers using concrete materials (Structured Cooperative Learning Groups).

Learning Objective 1: Combine two sets of concrete materials that represent fractions with mixed numbers.

Structured Peer Tutoring

Materials:
Teacher –

- appropriate number of concrete materials (area model) for each student pair.
- master copy of a learning sheet with multiple prompts: Each prompt 1.) identifies what sets of concrete materials should be combined (e.g. "two circles and three 'one-fourth' pieces plus one circle and one 'one fourth' piece.") and, 2.) includes the statement "Say the total."
- master copy of answer key for learning sheet: For each prompt the answer key identifies what the concrete materials should look like when combined –e.g. "four whole circles and no 'one-fourth' pieces." (*Pictures could also be drawn to cue students who have reading problems.)
- timer of clock for keeping time.

Students –

- each student in each pair has a learning sheet.
- each student pair has an appropriate set of concrete materials.
- each student pair has copy of answer key for coach to check player's responses.
- pencils for writing

Description:

Activity

Students work in pairs. One student is the "coach" for half of the period and one student is the "player." Students switch roles for the second half of the period. Each student has a learning sheet with appropriate prompts (See description of learning sheet under "Materials.") The coach gives the prompts from the learning sheet and the student uses concrete objects to respond. The coach checks the player's responses by examining the concrete objects and by consulting the answer key provided to each student pair. The coach awards two points for correct responses. If the player responds incorrectly, the coach provides appropriate feedback and then the player attempts the same set of prompts again. The coach awards the player one point if their second response is accurate. Tallies can be written on a separate sheet of notebook paper or in the margin beside each prompt. The teacher signals student pairs when it is time for students to switch roles. As students practice, the teacher circulates the room monitoring student behavior, providing specific corrective feedback, providing positive reinforcement, and prompting student thinking. Students total their points and turn in both their individual learning sheets and point sheets. The teacher reviews individual student learning sheets and point sheets to evaluate student understanding.

Structured Peer Tutoring Steps:

1.) Select pair groups and assign each pair a place to practice (try to match students of varying achievement levels if possible).
2.) Review directions for completing structured peer tutoring activity and relevant classroom rules. Practice specific peer tutoring procedures as needed (see step #4).
3.) Model how to perform the skill(s) within the context of the activity before students begin the activity. Model both what the coach does (e.g. reads the questions/prompts on the learning sheet; checks answers using answer key; provide corrective feedback; record points) and how the player responds (e.g. combining concrete materials; saying what the total represents).
4.) Divide the practice period into two equal segments of time. One student in each pair will be the player and will respond to the questions/prompts given by the coach, using concrete materials. The other student will be the coach and will say each question or prompt on the learning sheet. The coach will then write the response in the appropriate space on the player's learning sheet, check the answer key, and provide feedback regarding the player's response (e.g. positive verbal reinforcement for accurate responses and corrective feedback for inaccurate responses.) For inaccurate responses, the coach provides feedback and the player attempts the question a second time. The first response is crossed out and the second response is recorded. The coach records two points for correct responses on the first attempt and one point for correct responses on a second attempt.

5.) Provide time for student questions.
6.) Signal students to begin.
7.) Signal students when it is time to switch roles.
8.) Monitor students as they work in pairs. Provide positive reinforcement for both "trying hard," responding appropriately, and for students using appropriate tutoring behaviors. Also provide corrective feedback and modeling as needed.
9.) Students turn in learning sheets and point totals.
10.) Teacher reviews learning sheets/point totals to evaluate student understanding.

Learning Objective 3: Solve equations involving addition of fractions with mixed numbers using concrete materials.

Structured Cooperative Learning Groups – "Think-Pair-Share"

Purpose: to provide students multiple opportunities to practice solving equations involving addition of fractions with mixed numbers using concrete materials.

Materials:

Teacher –

- place concrete materials (area model) that represent wholes (e.g. circles) and one type of fractional part (e.g. one-fourth pieces) in zip-lock baggies. Include the appropriate number of each piece so they can solve the equations on their learning sheets. Choose the fractions your students need practice with (e.g. ½, ¼, 1/8) and decide how many student pairs will work with each type of fraction.
- develop a master copy for learning sheets that list equations involving addition of fractions with mixed numbers emphasizing specific fractional parts (e.g. "halves," "fourths," "eighths," etc.). Make appropriate number of copies for each student pair.
- Make "Sharing Solutions" Cue Sheet that list the steps student pairs use to share their solutions: 1.) say the equation they solved; 2.) represent the equation with concrete materials; 3.) add/combine materials; 4.) say solution.

Students –

- each student in each pair has learning sheet.
- each pair has appropriate baggie with concrete materials to solve equations on their learning sheet.
- each cooperative group has copy of "Sharing Solutions" Cue sheet.
- pencils for writing

Description:

Activity

Students are organized into groups of four to six students. Within each group, students are paired together. For each group, a different learning sheet that emphasizes different

fractional parts is given to each student pair. Student pairs "think" by responding to their respective learning sheets using concrete materials. After an appropriate period of time, the teacher signals student pairs to "share" their solutions with the rest of their cooperative group. Students in each pair take turns sharing at least two examples each. When sharing, students first identify the equation they solved, how they represented the equation with concrete materials, how they combined the materials to find a solution, say what the solution is. (*To cue students how to share their solutions, a sheet of paper labeled "Sharing Solutions" with these four steps listed can be placed in the middle of the table or floor where each cooperative group sits. A student in each cooperative group can be assigned the role of "Sharing Solutions Monitor" to help cue students as they share their solutions.) The teacher circulates the room as students work in pairs and as they share within their cooperative groups to monitor their work and to provide specific corrective feedback, positive reinforcement, and to prompt student thinking as they solve their equations. After an appropriate amount of time is provided for students to share within their cooperative groups, the teacher asks several pairs that solved equations for each type of fractional part to demonstrate their solutions to the whole class. The teacher provides specific corrective feedback, positive reinforcement, and prompts student thinking as the student pairs demonstrate their solutions. At the conclusion of the activity, students turn in their learning sheets and the teacher evaluates them to check for understanding.

Instructional Game/Structured Cooperative Learning Groups Steps:

1.) Provide explicit directions cooperative group activity including what you will do, what students will do, and reinforce any behavioral expectations for the game.
2.) Arrange students in cooperative groups and student pairs. Groups and student pairs should include students of varying skill levels.
3.) Assign roles to individual group members and explain them (e.g. Student pairs – one student is the concrete material "keeper" & one student is the "time keeper;" Cooperative Groups: "Sharing Solutions" monitor & "time keeper.")
4.) Distribute materials.
5.) Review/model appropriate cooperative group behaviors and expectations.
6.) Model one example of skill(s) (i.e. solving an equation involving addition of fractions with mixed numbers) within the context of the game.
7.) Provide opportunity for students to ask questions.
8.) Provide specific feedback/answer any additional questions as needed.
9.) Teacher circulates room, monitors students, provides specific corrective feedback, and positive reinforcement.
10.) Selected student pairs share their solutions to whole class.
11.) Students turn in individual learning sheets.
12.) Teacher reviews learning sheets and checks for understanding.

Instructional Phase 3: Evaluation of Student Learning/Performance (Initial Acquisition through Mastery/Maintenance)

1. Continuously Monitor & Chart Student Performance

Purpose: to provide you with continuous data for evaluating student learning and whether your instruction is effective. It also provides students a way to visualize their learning/progress.

Materials:

Teacher –
- appropriate prompts if they will be oral prompts
- appropriate visual cues when prompting orally

Student –
- appropriate response sheet/curriculum slice/probe
- graph/chart

Description:

Steps for Conducting Continuous Monitoring and Charting of Student Performance:

1.) Choose whether students should be evaluated at the receptive/recognition level or the expressive level.
2.) Choose an appropriate criteria to indicate mastery.
3.) Provide appropriate number of prompts in an appropriate format (receptive/recognition or expressive) so students can respond.

- Based on the skill, your students' learning characteristics, and your preference, the curriculum slice or probe could be oral in nature with visual cues (e.g. show two sets of concrete objects and then a concrete representation of their sum, say, "what is the total?", and students say the total; teacher/peer shows two sets of concrete objects and students/peer combine(s) their own sets at their desk), or a combination of written curriculum slices/probes and oral prompts with visual cues (e.g. show two sets of concrete objects and then a concrete representation of their sum, say, "what is the total?", and students circle the phrase that correctly identifies the total given several choices on a curriculum slice.).

4.) Distribute to students the curriculum slice/probe/response sheet/concrete materials.
5.) Give directions.
6.) Conduct evaluation.
7.) Count corrects and incorrects/mistakes (you and/or students can do this depending on the type of curriculum slice/probe used – see step #3).
8.) You and/or students plot their scores on a suitable graph/chart. A goal line that represents the proficiency (for concrete level skills, this should be 100% – 5 out of 5 corrects) should be visible on each students' graph/chart).

9.) Discuss with children their progress as it relates to the goal line and their previous performance. Prompt them to self-evaluate.
10.) Evaluate whether student(s) is ready to move to the next level of understanding or has mastered the skill at the abstract level using the following guide:

Concrete Level: demonstrates 100% accuracy (given 3 to 5 response tasks) over three consecutive days.

11.) Determine whether you need to alter or modify your instruction based on student performance.

2. Additional Assessment Activity Appropriate For This Math Skill/Concept

Purpose: to evaluate student conceptual understanding of the division process, with and without remainders.

Flexible Math Interview

Materials:

Teacher –
- a small notepad to write notes regarding particular student's understanding as you "interview" them.
- pencil for writing

Students -
- appropriate concrete objects

Description:
As students are working independently or in pairs, ask them to describe their solutions and how they arrived at them. Encourage students to both use concrete materials to do this as well as "talk about" what they are doing with their concrete materials. Note specific student misunderstandings/non-understandings on sheet of paper or individual student note-cards. Use the information collected to make instructional decisions for the whole class and for individual students.

Instructional Phase 4: Maintenance – Periodic Practice to Maintain Student Mastery of Skills

Purpose: to provide students with opportunities to maintain their level of mastery of adding fractions with mixed numbers using concrete materials.

Problem of the Day

Materials:

Teacher –

- a written prompt on the chalkboard, dry-erase board, or overhead projector (e.g. equation depicting addition of fractions with concrete materials) or a concrete example of some part of the process (e.g. sum of two sets of wholes and fractional parts).

Students -

- concrete materials if appropriate
- paper and pencil to record their responses if appropriate

Description:

Teacher presents a "problem of the day" that focuses on a particular skill or conceptual understanding of the addition of fractions with mixed numbers process. The problem can be written in nature (e.g. equation or word phrases that represent combing fractions with mixed numbers) where students respond with concrete objects or the problem can be represented with concrete materials. The "problem of the day" is displayed as students enter the room or as the period begins. Students are asked to "solve" the problem and provided necessary directions. After an appropriate amount of time, the teacher and the students "talk through" the problem and its solution. Students can individually describe how they approached the problem. Specific positive verbal reinforcement is provided by the teacher as well as specific feedback regarding misunderstandings students may have. Teacher notes students who seem to be having difficulty for the purpose of reviewing/re-modeling appropriate skills and concepts.