

## Instructional Plan

### Concrete Level

**Name of Math Skill/Concept:** Division Process and Division with Remainders

**Prerequisite Skills Needed:**

- multiplication process & facts
- grouping objects

**Learning Objectives:**

- 1.) Divide without remainders using concrete materials.
- 2.) Divide without remainders using concrete materials within a story problem context - measurement/"separating into groups" situations.
- 3.) Divide without remainders using concrete materials within a story problem context - partitive/"sharing" situations.
- 4.) Divide with remainders using concrete materials in measurement/"separating into equal groups" and partitive/"sharing" situations.

**Important Ideas for Implementing This Teaching Plan:**

- 1.) This teaching plan describes how you might teach both the division process as well as division with remainders using concrete materials. Depending on your grade level and your students' prior knowledge/experiences, you might teach both of the concepts/skills or you may only want to teach one of the two concepts/skills.
- 2.) If you are teaching the division process and division with remainders, students should achieve mastery of the division process at the concrete, representational/drawing, and the abstract levels of understanding *before* beginning instruction for division with remainders.
- 3.) If you are teaching only division with remainders and find your students are having difficulty, you might consider re-teaching the division process using the strategies described.
- 4.) At the concrete level of instruction, two division situations are taught including: 1.) Measurement/"Separating into Equal Groups" situations that require division by separating a set of objects into groups that contain a specified number of objects for the purpose of determining how many groups one can make; 2.) Partitive/"Sharing" that require division by "sharing" a total set of objects equally among a specified number individuals and determining how many objects each individual gets. These two "division situations" are both naturally occurring/authentic situations that require division.

5.) While this teaching plan incorporates the use of discrete counting objects, you might consider modeling and providing practice by using base-ten materials after students demonstrate understanding and mastery of basic division (e.g. one digit/two digit dividends divided by one/two digit divisors) through the abstract level. Base-ten materials can be very helpful when dealing with division problems that involve larger values (e.g. three digit dividends) and help reinforce understanding of place value.

### ***Instructional Phase 1: Initial Acquisition of Skill/Concept – Teacher Directed Instruction***

#### **1. Teach Skill/Concept within Authentic Context**

##### *Description:*

##### 1. Division Process without Remainders

Introduce the division process through naturally occurring contexts such as a child seeing how many of his friends he can take on a roller coaster ride and a teacher sharing pieces of candy for a class party. Use “tickets” made from construction paper and bags of candy, boxes of cupcakes, boxes of cookies to model “dividing.” (\*Remember, the total number of items should be such that they can be divided evenly.)

##### 2. Division with Remainders

Use the same contexts as described above but ensure the total number of items to be “divided” is such that their will be several left over after everyone has received an equal number. \*It may be helpful to first replicate the division process without remainders first, and then introduce the division situation with remainders.

#### **II. Build Meaningful Student Connections**

*Purpose:* to assist students to build meaningful connections between what they know about sharing things among their friends to the division process.

**Learning Objective 2: Divide without remainders using concrete materials within a story problem context – partitive/“sharing” situations.**

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##### *Materials:*

Teacher –

- a set of CD's or CD cases that can be divided evenly among two or three students.
- a bag of candy, box of cupcakes, or box of cookies, etc.
- a visual display with the word “divide” written on it.

##### *Description:*

1.) Link to students' prior knowledge of sharing things among their friends.

For Example:

I was at the Music Store yesterday and bought five new CD's for me and four of my friends. When I bought them, I then gave one CD to each of my friends and to myself. (Demonstrate this process by calling several students up and "acting the story situation out.) Can you think of times when you had a certain number of things and you shared those things with friends so that each friend got the same number of things? (Elicit student examples.)

2.) I dentify the skill students will learn: division

For Example:

Today, we are going to learn how to "divide." (Show the visual display with the word "divide" written.) When we divide, we do something very similar to sharing things with our friends. We're going to first learn how to divide using several different objects like \_\_\_\_\_ (e.g. candy, cupcakes, cookies). What are we going to learn to do today? (Point to the visual display of "divide" and elicit the response, "divide.") Yes, we are going to learn how to divide using several different kinds of materials.

3.) P rovide rationale/meaning for learning the division process

For Example:

Learning how to divide concrete objects will really come in handy. Once, you learn to divide a variety of objects well, you will then be able to share things with your friends, family, and others so that each person gets the same amount and doesn't feel badly that they received less than somebody else. (Call several students up and simulate this concept by giving them each different amounts of candy, cupcakes, or cookies and ask how each would feel.)

**Learning Objective 4: Divide with remainders using concrete materials in measurement/"separating into equal groups" and partitive/"sharing" situations.**

*Materials:*

Teacher –

- a bag of candy, box of cupcakes, or box of cookies, etc.
- a visual display with the words "division with remainders."

*Description:*

1.) L ink to student's prior knowledge of

For Example:

The past few days, you have been using concrete objects to divide. Let's do an example together. (Replicate the division process modeled when you divided candy/cupcakes/cookies evenly among the students in the class.) Did everybody get \_\_\_\_\_ number of candy pieces/cupcakes/cookies? (Elicit the response, "yes.") Everybody got the

equal number of candy pieces/cupcakes/cookies. Do I have any left in my bag/box? (Show students that there are no candy pieces left and elicit the response, "no.") That's right there are no candy pieces/cupcakes/cookies left.

2.) **I** dentify the skill students will learn:

For Example:

Today, you are going to learn about division/"divide" situations where we do have objects left over. We call this division with remainders. (Visually display the words "division with remainders.") What are we going to learn today? (Point to the display and elicit the response, "division with remainders.") Yes, we are going to learn division with remainders. What does "division with remainders mean? (Elicit the response, "when we divide and have objects left over." Cue students as needed.) Yes, division with remainders (Point to the visual display of "division with remainders.") means times when we divide and we have objects left over.

3.) **P** rovide rationale/meaning for learning how to divide in situations where there are remainders.

numbers.

For Example:

There will be times when you want to share things with your friends, like sharing cookies. (Call up four students to model this situation.) After I have given each friend the same number of cookies (Give each student cookies until you have several left over – 3 or less), I have several cookies left over. There isn't enough cookies left to give everybody one more cookie. (Show students the cookies you have left, count the number of total students there are to make this situation clear) If I gave some of my friends another cookie then others would not get as many. (Demonstrate this by "giving" one more cookie to some of the students.) If I did this, what would those who didn't get an extra one feel like? (Elicit the response, "they would feel bad/mad.") Yes. But, if I knew about division with remainders, then I could have told my friends that there will be some cookies left. That way nobody would have felt left out. Also, if I knew I would have several left over before I gave the cookies out, my friends and I could have decided what to do with the left over cookies before I gave them out. That way, everybody will feel good about things.

### **III. Provide Explicit Teacher Modeling**

*Purpose:* to provide students a clear model of the division process, with and without remainders, using concrete objects.

**Learning Objective 1: Divide without remainders using concrete materials.**

Materials:

Teacher –

- Language cards that have written on them the following phrases: "How many all together?"; "Separate into groups of two."; "How many groups?"
- concrete materials including discrete counting objects (e.g. beans, counting chips) and containers for grouping where the grouped counting chips are clearly visible to all students (i.e. small paper plates).
- A visible platform for demonstrating concrete objects where all students can see the concrete objects and your actions with them.

A. Break down the skill of the dividing without remainders using concrete materials.

- 1.) I identify the "total."
- 2.) I identify how many in each group.
- 3.) Place appropriate number of objects into groups until no objects are left.
- 4.) Count the number of groups.

B. Explicitly describe and model how to divide without remainders using concrete materials.

- 1.) Model how to identify the "total."
  - Introduce language card "How many all together?"
    - Think aloud
    - Read card aloud
    - Point to words and to concrete objects

For Example:

I have a group of beans here. I also have several cards with questions/directions. (Point to the group of eight beans and the cards.) These cards have questions or directions on them that will help me learn to divide using this group of beans. I'll turn over the first card, read it and then answer the question with my beans. (Turn the card over.) It says, "How many all together?" (Display the card and point to the words as you read them so that all students can see the words.)

- Model counting beans to find out how many you have all together.
  - Think aloud
  - Point to each objects as you count
  - Ask students - "How many?"

For Example:

Hmm, the card asks, "How many all together?" I can find that out by counting all of my beans. Help me count them. (Point to the beans as you count the beans with your students.) How many beans do I have all together? (Elicit the response, "eight.") Yes, I have eight beans all together.

2.) Model how to identify how many in each group.

- Introduce the language card "Separate into groups of two."
  - Think aloud
  - Read card aloud
  - Point to words

For Example:

Now, that I have answered my first card, I can turn my second card over and read it. (Turn the card over and read it aloud, pointing to the words as you read it. Then have your students read it aloud with you.)

What does the card say? (Elicit the response, "separate into groups of two.") Yes, I need to separate my eight beans into groups with two beans each.

- Model separating the group of eight beans into groups of two using paper plates.
  - Think aloud
  - Point to objects

For Example:

Hmm, how could I represent my groups? I know, I can use these small paper plates to put my groups of beans in. (Show students several paper plates.) I will place one plate down, and place two beans on it. (Place one plate down and then count aloud as you place two beans on the plate.) Ok, I have more beans left (Point to the remaining six beans.) so I can make another group. (Place a second paper plate on the table and then count aloud as you place two beans on the plate. \*Continue this process until you have no beans left.)

3.) Model how to find how many groups.

- Introduce language card "How many groups?"
  - Think aloud
  - Read card aloud
  - Point to words
  - Elicit student response

For Example:

Now that I have separated my eight beans into groups of two beans each (Point to each group of beans.), I can read my last card. (Turn the card over and read it aloud while pointing to the words as you read. Next, ask your students to read it with you.) What does the card say? (Elicit the response, "how many groups?") Yes, it asks, "How many groups?"

- Model counting the number of groups
  - Count groups aloud with students

For Example:

I can find the number of groups by simply counting the plates that contain groups of two beans. Do that with me. (Count aloud the number of groups with your students. Point to each plate as you count.) How many groups of two beans do we have? (Elicit the response, "four.") Great! We have four groups of two beans.

- Prompt student thinking about the "grouped" beans and the total number of beans you started with.
  - Point to each group
  - Count objects

For Example:

Now that we have put our eight beans into four groups of two, how many total beans do we have in all the groups? (Elicit the response, "eight." Cue students as needed by pointing to each group and counting the beans.) Yes, we have eight beans total in all the groups. How many beans did we have to start with? (Elicit the response, "eight.") Yes, we had eight beans to start with. So, we didn't change the number of total beans, we just put them into four groups of two beans.

4.) Repeat steps 1-3 using at least two more examples.

**Learning Objective 2: Divide without remainders using concrete materials within a story problem context - measurement/"separating into equal groups" situations.**

*Materials:*

Teacher -

- visual display of story problem for division; number phrases representing dividend/total and divisor are color-coded.
- concrete materials that represent objects in story problem (e.g. tickets, candy pieces, string, small paper plates).
- visual displays for the words "dividend," "divisor," and "quotient." " Color-code the "dend" in "dividend" and the "sor" in "divisor" to match the corresponding number phrases in the story problem.
- markers/chalk for writing.

A. Break down the skill of dividing without remainders within a story problem context - measurement/"separating into equal groups" situations.

- 1.) Introduce story problem.
- 2.) Read the story problem aloud and then have students read it with you.
- 3.) Teach how to find the important information in the story problem.
- 4.) Represent the important information with concrete materials.
- 5.) Model the division process by “dividing” the objects into equal groups on plates.
- 6.) Model how to solve the story problem with the concrete objects.
- 7.) Replicate division process with a variety of concrete objects.

B. Explicitly describe and model how to divide without remainders within the context of a story problem - measurement/“separating into equal groups” situations.

- 1.) Introduce the story problem. Highlight/cue the important information in the story problem by color-coding the “dividend” and the “divisor.”
  - color-code dividend and divisor
  - make visible to all students

For Example:

“Jeremy has **twenty tickets** for the “Master-Blaster” roller coaster ride. It costs **two tickets** to go on the ride and Jeremy wants to take his friends with him on the ride. How many of Jeremy’s friends including himself can go on the ride?”

- 2.) Read the story problem aloud and then have students read it with you.
  - Point to words
  - Prompt student thinking about context

For Example:

I have a story problem here. (Point to the visual display of the story problem.) We are going to use what we have learned about separating objects into groups to solve this story problem. This will help us to learn how to “divide” even better. (Point to the visual display of the word “divide.”) I’ll read the story problem first and then I’ll ask you to read it with me a second time. (Read the story problem aloud first and then invite students to read it with you a second time.)

- 3.) Teach how to find the important information in the story problem.

- Link process of finding important information of addition, subtraction, and multiplication story problems to division story problems

For Example:



Ok, we have a story problem. Hmm, I remember that when we had story problems for addition, subtraction, and multiplication, there was important information in the story problem that helped me solve it. I think the same will be true for this division story problem.

- Model finding what you are solving for.
  - think aloud
  - skim story problem with finger
  - point to and circle question mark
  - underline question

For Example:

I remember that one thing I need to do when finding the important information in a story problem is to find what I am solving for. Hmm, I also remember that I can look for a question mark to help me find the question that tells me what I am solving for. That is one sign or cue that tells me where to find what I am solving for. Aha, here's the question mark. When I find it, I can circle it to help me remember where it is. (Circle the question mark.) I now need to read the question, because this will tell me what I am solving for. I'll do that now. (Read the question.) I can underline the question. That will also help me remember what I am solving for. (Underline the sentence that is the question.) What am I solving for? (Point to the question and elicit the response, "how many of his friends including himself can go on the ride?") Yes, I need to find out how many of his friends including himself can go on the ride.

- Model finding the number phrases that represent the "dividend/total" and the "divisor."
  - Think aloud
  - Point to words as you read
  - Circle important information
  - Review circled information
  - Question students

For Example:

Now that I know what I am solving for, I also remember story problems usually have number phrases that represent important information. Let me see if I can find the important information by looking for number phrases. I can do that by reading each sentence aloud and then asking myself, "is there a number phrase in this sentence?" I'll do that now. (Model reading each sentence and then asking yourself, "is there a number phrase in this sentence?" Circle the number phrases "twenty tickets" and "two tickets" as you come to them.) Ok, I've read all the sentences in the story problem and I found two number phrases. What is my first number phrase? (Elicit the response, "twenty tickets.") Yes. Jeremy has twenty tickets. What is my second number phrase? (Elicit the response, "two tickets.") Good. It costs two tickets for a person to ride the ride. Are there any more number phrases? (Elicit the response, "no.")

- Model deciding if all the important information is identified.
  - Think aloud
  - Point to highlighted information
  - Prompt student thinking

For Example:

Now, I need to decide whether I have all the important information identified. Did I find what I am solving for? Well, yes I did. (Point to the underlined question and the circled question mark.) What am I solving for? (Elicit the response, "how many can go on the ride?") How did you know that this is what I am solving for? (Elicit the response, "because you underlined it/circled the question mark" and/or "it is the question.") Good. I know this is what I am solving for because I underlined it and circled the question mark. I know that questions usually help me to know what I am solving for. Now that I am sure that I know what I am solving for, what other important information did I need to find? (Point to the circled number phrases, and elicit the response, "number phrases.") Yes, I found the number phrases. Did I find all of them? (Elicit the response, "yes.") How do I know I found all of them? (Elicit the response, "you read each sentence aloud and asked yourself, 'is there a number phrase in this sentence?'" Excellent job of remembering how I made sure I found all of the number phrases! You are right, I read each sentence in the story problem and then asked myself, "is there a number phrase in this sentence?") Now that I know I have found what I am solving for and all the important information in the story problem, I am ready to solve the story problem.

4.) Represent the "important information" in the story problem with concrete materials.

- Model the dividend with concrete objects (i.e. twenty "tickets" made from construction paper or tag board) and explicitly relate the twenty tickets to the story problem. Introduce the term "dividend."
  - Point to circled phrase
  - Prompt student thinking
  - Count objects aloud/say total
  - Display "dividend" language card and say "dividend" aloud
  - Students say "dividend"

For Example:

Now, I am going to use my tickets to help me figure out how to solve this story problem. How many tickets does the story problem say Jimmy has? (Elicit the response, "twenty." Point to the phrase, "**twenty tickets**" to emphasize.) Ok, here are the tickets. I'll count out twenty tickets. (Count aloud the twenty tickets.) Invite students to count aloud with you.) How many tickets do I have? (Elicit the response, "twenty.") Why did I count out twenty tickets? (Elicit the response, "because that is how many tickets in the story problem.") Great. Now, "twenty" is the total number of tickets I have. There is a special name we call this total number

when we are doing division. It is called the "dividend." (Show the visual display of the word "dividend." Color-code the "dend" in blue to match the phrase "twenty tickets" in the story problem. This will help students with visual processing difficulties to differentiate the meaning of the word "dividend" from the word "divisor" that will be introduced next.) What do we call the total number of objects in division? (Elicit the response, "dividend.") What is our dividend in this story problem? (Elicit the response, "twenty." Point to the number phrase "twenty tickets" and then to the group of twenty tickets you just counted out.) Great! We have a total of twenty tickets and this group of twenty tickets is called the "dividend." (Point to the word "dividend.")

- Model the divisor with concrete objects and introduce the term "divisor."
  - Point to circled phrase
  - Prompt student thinking
  - Count objects on plates aloud/say total
  - Display "divisor" language card and say "dividend" aloud
  - Students say "divisor"

For Example:

Now that I have counted out the total number of tickets, or my "dividend" (Point to the visual display "dividend."), I now need to represent my other number phrase. (Point to "two tickets.") What do I also need to represent with concrete objects? (Elicit the response, "two tickets.") Yes, I need to represent two tickets. I can do that by counting two of the tickets I have and putting them over here. (Count two tickets and place them in a group away from the rest of the tickets.) I have something else that can help me separate the two tickets needed to ride the ride. I can use the paper plate. (Show the paper plate and then place the two tickets on the paper plate.)

Now, I want to introduce you to another new word. The two tickets we just counted and placed on the paper plate are called the "divisor" when we are dividing. (Point to the visual display "divisor." \*Color-code the "sor" red to match the color used for the phrase "two tickets." This will help students with visual processing difficulties to differentiate the meaning of the word "divisor" from the word "dividend.") What do we call the two tickets we counted out and placed over here? (Point to the tickets and then point to the word "divisor" to elicit the response, "divisor.") Yes, a name for the two tickets is "divisor." (Point to the word "divisor.")

5.) Model the division process by "dividing" the objects (separating them into equal groups - with string or by dealing objects evenly among small paper plates). Explicitly relate this action to the story context and relevant important information. Re-emphasize the language "dividend" and "divisor."

- Point to words as you read
- Prompt student thinking
- count objects aloud

For Example:

We have all of the important information in the story problem represented with our group of twenty tickets. We know we can represent the two tickets it takes to ride the roller coaster by counting them aloud and putting them in a separate place using a paper plate. Now, let's use them to solve the story problem. We can do this by re-reading the story problem and acting it out. Let's read the first sentence together. (Point to the first sentence and read it aloud with your students. Point to each word as you read.) Ok, we know that Jeremy has twenty tickets. (Point to the highlighted number phrase "twenty tickets.") The twenty tickets are here. (Point to your group of twenty tickets.) We have already counted them so I know we have twenty tickets. Now, I'll read the next sentence. (Point to the second sentence and read it aloud with your students. Point to each word as you read.) This sentence tells us that it takes two tickets to ride the ride. Well, we can represent each group of two tickets by counting two tickets and placing them on a paper plate. (Point to the number phrase, "two tickets" and then to a paper plate.) It says that Jeremy wants to take his friends with him on the ride (Point to the appropriate phrase.). How many tickets will it cost for each person to ride the ride? (Point to "two tickets" and elicit the response, "two."). Yes, it costs two tickets for each person. I can deal out two tickets per paper plate until I have given out all of the tickets. (Demonstrate this process, by laying the paper plates out in a line or several lines and then dealing out two tickets per plate until all tickets are dealt and each plate has two tickets.)

6.) Model how to solve the story problem with the concrete objects.

- Think aloud
- Point to words
- Cue highlighted information with finger
- Prompt student thinking
- Display "quotient" language card

For Example:

Alright, we have read through most of the story problem and we have acted it out with tickets and paper plates. We still need to read the last sentence. What does the last sentence tell us? (Elicit the response, "what we are solving for.") Yes. How do you know that? (Elicit the response, "because it is a question.") Good. Questions in story problems usually tell me what to solve for. (Point to the underlined question.) Let's read it aloud. (Read the question aloud with students. Point to the words as you read.) What is it we need to solve for? (Point to the underlined question, and elicit the response, "how many of Jeremy's friends including himself can go on the ride?") Yes, we need to decide how many children can go on the ride. Well, we can figure this out by looking at what I have already done with my tickets and paper plates. First, I counted out twenty tickets because that was my total or dividend (Point to "twenty tickets" in the story problems and then point to the word "dividend" word card.) Next, I read that I needed to group the tickets by two because the story problem says it takes two tickets to ride the roller coaster ride. "Two tickets" is my divisor. (Point to "two

**tickets**" in the story problem and then point to the **"divisor"** word card.) Now, let's look again at the question and solve the story problem. (Read aloud the question and prompt student thinking about what you need to find out.) To find out how many children got tickets to go on the ride, I can simply count the number of plates that have two tickets on them. Each plate represents a child who got two tickets for the roller coaster ride. (Count the plates aloud.) How many children got tickets? (Elicit the response, "ten.") Yes, ten children got tickets. I know this because each plate has two tickets on it and there are ten plates. Let's count the plates one more time. (Count aloud the plates with your students. Point to the plates as you count.) So, what is the solution to the question, "How many of his friends and Jeremy can go on the ride?" (Point to the underlined question and elicit the response, "ten.") Great! Ten children, Jeremy and nine of his friends, got two tickets and therefore those ten children can go on the "Master-Blaster" roller coaster.

Now that we know our solution to this division problem, I want to introduce you to a name we call the solution to a division problem. We call it the "quotient." (Display the visual "quotient.") We call the solution to a division problem the quotient when we have a total or dividend (Point to the visual display of **"dividend."**) and we have a divisor (Point to the visual display of **"divisor."**). What do we call the solution to a division problem when we are given the dividend and the divisor? (Point to the appropriate visual display and elicit the response, "quotient."). Yes, we call the solution "quotient." (Point to the visual display "quotient."). What is our quotient? (Elicit the response, "ten.") Yes, our quotient is "ten" and it represents the number of children that can go on the "Master-Blaster" roller coaster.

\*When introducing the terms, dividend, divisor, and quotient, it can be helpful for the future to line the word cards up in a fashion that represents a typical division equation (e.g. "dividend divisor quotient" or "dividend" "divisor" "quotient")

Doing this will be a helpful cue as students begin setting up and solving division equations from given division situations.

7.) Replicate division process with measurement/"separating into groups" situations with a variety of counting objects and containers by repeating steps 1-7 (e.g. trading chips, unifix cubes, beans and containers that allow students to clearly see the number of counting objects in them – paper plates, cut out pieces of tag-board, CD disk covers, etc.). Use examples that include smaller numbers of objects.

**Learning Objective 3: Divide without remainders using concrete materials within a story problem context – *partitive/"sharing" situations.***

*Materials:*

Teacher –

- visual display of story problem for division; number phrases representing dividend/total and divisor are color-coded.
- concrete materials that represent objects in story problem (e.g. tickets, candy pieces, string, small paper plates).
- visual displays for the words "dividend," "divisor," "quotient." " Color-code the "dend" in "dividend" and the "sor" in "divisor" to match the corresponding number phrases in the story problem.
- markers/chalk for writing.

A. Break down the skill of dividing without remainders using concrete materials within a story problem context - partitive/"sharing" situations.

- 1.) Introduce story problem.
- 2.) Read the story problem aloud and then have students read it with you.
- 3.) Teach how to find the important information in the story problem.
- 4.) Represent the important information with concrete materials.
- 5.) Model the division process by "dividing" the objects evenly among a specified number of groups.
- 6.) Model how solve the story problem with the concrete objects.
- 7.) Replicate division process with a variety of concrete objects.

B. Explicitly describe and model how to divide without remainders using concrete materials within a story problem context - ***partitive/"sharing" situations.***

- 1.) Introduce the story problem. Highlight/cue the important information in the story problem by color-coding the "dividend" and the "divisor."
  - color-code dividend and divisor
  - make visible to all students

For Example:

"Mrs. Crouch has a bag of 50 candy pieces. She wants to give each of her 25 students the same number of candy pieces. How many candy pieces should each student get?"

- 2.) Read the story problem aloud and then have students read it with you.
  - Point to words
  - Prompt student thinking about context

For Example:

I have a story problem here. (Point to the visual display of the story problem.) This story problem will help us learn how to “divide.” (Point to the visual display of the word “divide.”) I’ll read the story problem first and then I’ll ask you to read it with me a second time. (Read the story problem aloud first and then invite students to read it with you a second time.)

3.) Teach how to find the important information in the story problem.

- Link process of finding important information of addition, subtraction, and multiplication story problems to division story problems

For Example:

Ok, we have a story problem. Hmm, I remember that when we had story problems for addition, subtraction, and multiplication, that there was important information that was in the story problem that helped me solve the story problem. I think the same will be true for this division story problem.

- Model finding what you are solving for.
  - think aloud
  - skim story problem with finger
  - point to and circle question mark
  - underline question

For Example:

I remember that one thing I need to do when finding the important information in a story problem is to find what I am solving for. Hmm, I also remember that I can look for a question mark. That is one sign or cue that tells me where to find what I am solving for. Aha, here is the question mark. When I find it, I can circle it to help me remember where it is. (Circle the question mark.) I now need to read the question, because this will tell me what I am solving for. I’ll do that now. (Read the question.) I can underline the question. That will also help me remember what I am solving for. (Underline the sentence that is the question.) What am I solving for? (Point to the question and elicit the response, “how many candy pieces should each student get?”) Yes, I need to find out how many candy pieces each student should get.

- Model finding the number phrases that represent the “dividend/total” and the “divisor.”
  - Think aloud
  - Point to words as you read
  - Circle important information
  - Review circled information
  - Question students

For Example:

Now that I know what I am solving for, I also remember story problems usually have number phrases that represent important information. Let me see if I can find the important information by looking for number

phrases. I can do that by reading each sentence aloud and then asking myself, "is there a number phrase in this sentence?" I'll do that now. (Model reading each sentence and then asking yourself, "is there a number phrase in this sentence?" Circle the number phrases "50 candy pieces" and "25 students" as you come to them.) Ok, I've read the all the sentences in the story problem and I found two number phrases. What is my first number phrase? (Elicit the response, "fifty candy pieces.") Yes. Mrs. Crouch has a bag with fifty candy pieces. What is my second number phrase? (Elicit the response, "twenty-five students.") Good. Mrs. Crouch has twenty-five students who she wants to give an equal number of candy pieces. Are there any more number phrases? (Elicit the response, "no.")

- Model deciding if all the important information is identified.
  - Think aloud
  - Point to highlighted information
  - Prompt student thinking

For Example:

Now, I need to decide whether I have all the important information identified. Did I find what I am solving for? Well, yes I did. (Point to the underlined question and the circled question mark.) What am I solving for? (Elicit the response, "how many candy pieces should each student get?") How did you know that this is what I am solving for? (Elicit the response, "because you underlined it/circled the question mark" and/or "it is the question.") Good. I know this is what I am solving for because I underlined it and circled the question mark. I know that questions usually help me to know what I am solving for. Now that I am sure that I know what I am solving for, what other important information did I need to find? (Point to the circled number phrases, and elicit the response, "number phrases.") Yes, I found the number phrases. Did I find all of them? (Elicit the response, "yes.") How do I know I found all of them. (Elicit the response, "you read each sentence aloud and asked yourself, 'is there a number phrase in this sentence?'" Excellent job of remembering how I made sure I found all of the number phrases! You are right, I read each sentence in the story problem and then asked myself, "is there a number phrase in this sentence?") Now that I know I have found what I am solving for and all the important information in the story problem, I am ready to solve the story problem.

4.) Represent the "important information" in the story problem with concrete materials.

- Model the dividend with concrete objects and explicitly relate the fifty candy pieces to the story problem. Introduce the term "dividend."
  - Point to circled phrase
  - Prompt student thinking
  - Count objects aloud/say total
  - Display "dividend" language card and say "dividend" aloud
  - Students say "dividend"



### For Example:

Now, I am going to use my candy pieces to help me figure out how to solve this story problem. How many candy pieces does the story problem say Mrs. Crouch had in her bag? (Elicit the response, "fifty." Point to the phrase, "50 candy pieces" to emphasize.) Ok, here is my bag of candy. I'll count out fifty candy pieces. (Count aloud the fifty pieces of candy. Invite students to count aloud with you.) How many candy pieces do I have? (Elicit the response, "fifty.") Why did I count out fifty candy pieces? (Elicit the response, "because that is how many candy pieces in the story problem.") Great. Now, "fifty" is the total number of candy pieces I have. There is a special name we call this total number when we are doing division. It is called the "dividend." (Show the visual display of the word "dividend." Color-code the "dend" in blue to match the phrase "50 candy pieces" in the story problem. This will help students with visual processing difficulties to differentiate the meaning of the word "dividend" from the word "divisor" that will be introduced next.) What do we call the total number of objects in division? (Elicit the response, "dividend.") What is our dividend in this story problem? (Elicit the response, "fifty." Point to the number phrase "50 candy pieces" and then to the group of fifty candy pieces you just counted out.) Great! We have a total of fifty candy pieces and this group of fifty candy pieces is called the "dividend." (Point to the word "dividend.")

- Model the divisor with concrete objects and introduce the term "divisor."
  - Point to circled phrase
  - Prompt student thinking
  - Count objects on plates aloud/say total
  - Display "divisor" language card and say "dividend" aloud
  - Students say "divisor"

### For Exam

Now that I have counted out the total number of candy pieces, or my "dividend" (Point to the visual display "dividend."), I now need to represent my other number phrase. (Point to "25 students.") What do I also need to represent with concrete objects? (Elicit the response, "twenty-five students.") Yes, I need to represent Mrs. Crouch's twenty-five students. Hmm, I wonder how I can represent twenty-five students? Well, one way to do that is to use all of you! You are the students in my class. Let's count each of you to be sure there are twenty-five of you. (Have students count themselves aloud by one student beginning and each student counting on until all have counted.) How many students are there? (Elicit the appropriate response.) Now, I could "divide" the fifty candy pieces by giving them to you directly, but that will take some time to do and I know another way to solve the story problem with paper plates. (Hold up some paper plates.) I can use one paper plate to represent each of you. There are twenty-five of you, so I need how many plates? (Elicit the response, "twenty-five.") Yes, I need to count out twenty-five plates because there are twenty-five of you. I need twenty-five plates because each plate represents one student each. Let's count out twenty-five plates. (Count out twenty-five plates as students count aloud with you.) Ok, how many plates do I have? (Elicit the response,

"twenty-five.") Good. And who do these plates represent? (Elicit the response, "the twenty-five students in this class.") Yes, each plate represents one of you. How did I know I needed twenty-five plates? (Elicit the response, "because the story problem said there were twenty-five students." Point to the phrase "25 students" to emphasize.). Now that we know how many students we are going to divide the candy pieces among, I want to introduce you to another new word. The twenty-five students/plates is called the "divisor" when we are dividing. (Point to the visual display "divisor." \*Color-code the "sor" red to match the color used for the phrase "25 students." This will help students with visual processing difficulties to differentiate the meaning of the word "divisor" from the word "dividend.") What do we call the twenty-five students represented by these plates? (Elicit the response, "divisor." Point to the word "divisor" to emphasize.) Yes, we have a name for the twenty-five students we are representing with plates is "divisor." (Point to the word "divisor.")

5.) Model the division process by "dividing" the objects evenly among a specified number of groups.

(Represent groups with string or small paper plates). Explicitly relate this action to the story context and relevant important information. Re-emphasize the language "dividend" and "divisor."

- Point to words as you read
- Prompt student thinking
- count objects aloud

For Example:

Now that we have all of the important information in the story problem represented with candy pieces and paper plates, let's use them to solve the story problem. We can do this by re-reading the story problem and acting it out. Let's read the first sentence together. (Point to the first sentence and read it aloud with your students. Point to each word as you read.) Ok, we know Mrs. Crouch has a bag of fifty candy pieces. (Point to the highlighted number phrase "fifty candy pieces.") The fifty candy pieces are here. (Point to your group of candy pieces.) We have already counted them so I know we have fifty candy pieces. Now, I'll read the next sentence. (Point to the second sentence and read it aloud with your students. Point to each word as you read.) This sentence tells us that there are twenty-five students and that Mrs. Crouch wants to give each student an equal number of candy pieces. Well, we have the twenty-five students represented with the twenty-five paper plates. (Point to the number phrase, "25 students" and then to the twenty-five paper plates.) It says that Mrs. Crouch wants to share all of her fifty pieces of candy (Point to the appropriate phrase.) among her twenty-five students (Point to the appropriate phrase.) so that every student gets an equal number. Well, we can do that with our candy pieces and our paper plates. I can deal out one candy piece per paper plate until I have given out all of the candy pieces. (Demonstrate this process, by laying the paper plates out in a line or several lines and then dealing out one candy piece at a time per plate until all candy pieces are dealt and each plate has two candy pieces.)

6.) Model finding the solution to the story context with the concrete objects.

- Think aloud
- Point to words
- Cue highlighted information with finger
- Prompt student thinking
- Display "quotient" language card

For Example:

Alright, we have "acted out" the story problem. Now we need to solve the problem. What is it we need to solve for? (Point to the underlined question, and elicit the response, "how many candy pieces should each student get?") Yes, we need to decide how many pieces of candy each student should get. Well, we've already divided the fifty candy pieces among the twenty-five plates that represent the children. How many candy pieces did each student get? (Elicit the response, "two.") Yes, each student gets two pieces of candy. I know this because each plate has two pieces of candy on it. To solve this story problem, I can look at one plate and count the total number of candy pieces on it. (Point to one plate and count aloud the candy pieces.) So, what is the solution to the question, "How many candy pieces should each student get?" (Point to the underlined question and elicit the response, "two.") Great! Each student should get two pieces of candy. I know this because I dealt all fifty pieces of candy evenly among the twenty-five plates and each plate has two pieces of candy on it. (Point to one plate and count out the two candy pieces.) Now that we know our solution to this division problem, I want to introduce you to a name we call the solution to a division problem. We call it the "quotient." (Display the visual "quotient.") We call the solution to a division problem the quotient when we have a total or dividend (Point to the visual display of "dividend.") and we have a divisor (Point to the visual display of "divisor."). What do we call the solution to a division problem when we are given the dividend and the divisor? (Point to the appropriate visual display and elicit the response, "quotient."). Yes, we call the solution "quotient." (Point to the visual display "quotient."). What is our quotient? (Elicit the response, "two.") Yes, our quotient is "two" and it represents the number of candy pieces each student should get.

\*When introducing the terms, dividend, divisor, and quotient, it can be helpful for the future to line the word cards up in a fashion that represents a typical division equation (e.g. "dividend divisor quotient" or "dividend" "divisor" "quotient")

Doing this will be a helpful cue as students begin setting up and solving division equations from given division situations.

7.) Replicate division process with a variety of counting objects and containers by repeating steps 1-7 (e.g. trading chips, unifix cubes, beans and containers that allow students to clearly see the number of counting

objects in them – paper plates, cut out pieces of tag-board, CD disk covers, etc.). Use examples that include smaller numbers of objects.

**Learning Objective 4: Divide with remainders using concrete materials in measurement/"separating into equal groups" and partitive/"sharing" situations.**

*Materials:*

Teacher –

- visual display of story problem for division; number phrases representing dividend/total and divisor are color-coded.
- concrete materials that represent objects in story problem (e.g. tickets, candy pieces, small paper plates).
- visual displays for the words "dividend," "divisor," "quotient," and "remainder." Color-code the "dend" in "dividend" and the "sor" in "divisor" to match the corresponding number phrases in the story problem.
- markers/chalk for writing.

A. Break down the skill of division with remainders using concrete materials in measurement/"separating into equal groups" and partitive/"sharing" situations.

- 1.) Introduce story problem.
- 2.) Read the story problem aloud and then have students read it with you.
- 3.) Teach how to find the important information in the story problem.
- 4.) Represent the important information with concrete materials.
- 5.) Model "dividing" the objects in at least one of two ways: 1.) dealing objects one-by-one among containers (e.g. small paper plates) until there are fewer objects left over than there are plates – **partitive/"sharing"**; 2.) circling groups of objects based on the "divisor" until there are left over objects that are too few to be grouped – **measurement/"separating into equal groups"**. Explicitly relate this action to the story context and relevant important information.
- 6.) Model how to find the solution to the story context with the concrete objects and explicitly naming the "left over" objects as the "remainder."
- 7.) Replicate division process with remainders using a variety of discrete counting objects.

B. Explicitly describe and model how to divide with remainders using concrete materials in measurement/"separating into equal groups" and partitive/"sharing" situations.

**Partitive/"Sharing"**

1.) Introduce the story problem. Highlight/cue the important information in the story problem by color-coding the "dividend" and the "divisor."

- color-code dividend and divisor
- make visible to all students

For Example:

"Mrs. Crouch has a bag of 55 candy pieces. She wants to give each of her 25 students the same number of candy pieces. How many candy pieces should each student get?"

2.) Follow the same process as described for steps 2-5 of teaching the Division Process. \*Link the story context to the one used to introduce the Division Process. Prompt students to compare the two story problems and decide what is the same and what is different (i.e. the total/dividend).

6.) Model finding the left over objects and naming the left over objects the "remainder."

- Model finding the left over objects.
  - Think aloud
  - Point to objects as you count aloud
  - Prompt student thinking

For Example:

Hmm, I have five candy pieces left over. If I gave these candy pieces to five students, how many pieces would those five students get? (Deal the remaining pieces among five plates, point to the candy pieces on each plate, and elicit the response, "three.") Yes, these students would get 'one, two, three' pieces of candy (Point to each piece of candy as you count aloud). How many would the other twenty-students have? (Point to a plate with two candy pieces and elicit the response, "two.") Yes, they would have only two candy pieces. Would all the students have an equal number of candy pieces? (Elicit the response, "no.") No they wouldn't. Five students would have three pieces (Point to a plate with three candy pieces) and the remaining twenty students would get only two (Point to a plate with two candy pieces). Since, Mrs. Crouch wanted each student to have an equal number of candy pieces, what should she do? (Elicit the response, "put the remaining five candy pieces back in the bag.") Yes, if she sees she has some candy pieces left over and there are not enough to give every student one, then she could put the left over, or remaining candy pieces back in her bag. (Model taking the five remaining candy pieces off of the five plates and put them in the bag. How many candy pieces does each of the twenty-five students have now? (Point to all of the plates and elicit the response, "two.") Yes, all twenty-five students have two candy pieces each.

- Model naming the left over objects the "remainder."
  - Point to/identify left over objects

- Display "remainder" language card
- Prompt student thinking

For Example:

Now, let's look at the five remaining pieces of candy again. (Take the five pieces out of the bag and lay them in a group beside the plates.) These five pieces of candy are "left over" or "remaining" because there are not enough pieces to give all twenty-five students another piece. There is a special name for these left over pieces. We recall the "left over" or remaining pieces the "remainder." (Display the card with "remainder" on it.) What do we call the pieces that are "left over"? (Elicit the response, "the remainder.") Yes, we call the left over pieces the "remainder." (Point to the five pieces of candy and then to the card with the word "remainder.") These five pieces of candy are called the remainder. What is the remainder? (Elicit the response, "the five pieces of candy.") Why do we call these pieces of candy the remainder? (Elicit the response, "because there aren't enough to give each student another piece of candy.") Excellent thinking. Because there aren't enough pieces of candy to give all twenty-five students another piece, then it is called the remainder (Point to the card with the word "remainder.").

7.) Repeat the steps at least two more times with different discrete counting objects (e.g. unifix cubes, counting chips, etc.).

**Measurement/"Separating into Equal Groups"**

1.) Introduce story problem. Highlight/cue the important information in the story problem by color-coding the "dividend" and the "divisor."

- color-code dividend and divisor
- make visible to all students

For Example:

"Jeremy has **twenty-one tickets** for the "Master-Blaster" Roller coaster ride. It costs **two tickets** to go on the ride and Jeremy wants to take his friends with him on the ride. How many of his friends including himself can go on the ride?"

2.) Follow the same process as described for steps 2-5 of teaching the Division Process. \*Link the story context to the one used to introduce the Division Process. Prompt students to compare the two story problems and decide what is the same and what is different (i.e. the total/dividend).

6.) Model finding the left over objects and naming the left over objects the "remainder."

- Model finding the left over objects

- Think aloud
- Point to objects as you count aloud
- Prompt student thinking

For Example:

Hmm, I have one ticket left over. (Point to the left over ticket) If Jeremy gave this ticket to one more of his friends, would that friend be able to go on the roller coaster ride? (Elicit the response, "no.") You're right! The friend could not go on the roller coaster because he would need two tickets to go and there is only one ticket left. So, how many of Jeremy's friends including himself can go on the ride? (Elicit the response, "ten.") Yes, ten children have two tickets each (Point to the plates and count them aloud.), so ten children can go on the Master Blaster Roller coaster ride. Since each of the ten children has two tickets each, then there are ten "groups" of two tickets (Point to each group of tickets and count each group of tickets.) How many groups of two tickets are there? (Elicit the response, "ten.") Yes, there are ten groups of two tickets. How many tickets are left over? (Point to the "left over" ticket and elicit the response, "one.") Yes, there is one ticket left over.

- Model naming the left over objects the "remainder."
  - Point to/I identify left over objects
  - Display "remainder" language card
  - Prompt student thinking

For Example:

Now, lets look at the remaining ticket. (Point to the "left over" ticket.) This ticket is "left over" or "remaining" because it takes two tickets to ride the roller coaster. Just like we learned when we were dividing candy pieces, there is a special name for objects that are "left over." Who remembers what we call what is "left over" or "remaining?" (Elicit the response, "remainder.") Yes, we call it the remainder. Where do is the card that reminds us what this name is? (Encourage students to point out where the "remainder" word card is displayed and point to the word on the card.) What do we call the "left over" or remaining ticket? (Hold up the ticket, point to the "remainder" word card and elicit the response, "remainder.") Yes, we recall this "left over" or remaining ticket the "remainder." Why do we this ticket the remainder? (Elicit the response, "because it takes two tickets to go on the roller coaster.") Excellent thinking. Because there aren't enough tickets for another student to go on the roller coaster, that left over ticket is called the remainder (Point to the card with the word "remainder.").

7.) Repeat the steps at least two more times with different discrete counting objects (e.g. unifix cubes, counting chips, etc.).

#### IV. Scaffold Instruction

\*The steps for scaffolding your instruction are the same for each concept you have explicitly modeled and with each Division Situation you teach (Measurement/"Separating into Equal Groups," & Partitive/"Sharing"). This teaching plan provides you a detailed example of scaffolding instruction for one of the concepts modeled during Explicit Teacher Modeling - Division with Remainders Using Partitive/"Sharing" Situations, emphasizing the essential steps in the problem-solving process. You should scaffold your instruction with each skill/concept you model.

*Purpose:* to provide students the opportunity to build their initial understanding of the division process, with and without remainders, and to provide you the opportunity to evaluate your students' level of understanding after your initial modeling of these skills.

#### Learning Objective 4: Division with Remainders Using Partitive/"Sharing" Situations

##### *Materials:*

Teacher -

- visual display of story problem for division; number phrases representing dividend/total and divisor are color-coded.
- concrete counting objects (e.g. unifix cubes, counting chips, small paper plates).
- concrete objects/containers for grouping (e.g. paper plates, note cards, pieces of string)
- visual displays for the words "dividend," "divisor," "quotient," and "remainder." Color-code the "dend" in "dividend" and the "sor" in "divisor" to match the corresponding number phrases in the story problem.
- markers/chalk for writing.

Students -

- concrete counting objects (e.g. unifix cubes, counting chips, small paper plates).
- concrete objects/containers for grouping (e.g. paper plates, note cards, pieces of string)

##### *Description:*

#### 1.) Scaffold Using a High Level of Teacher Direction/Support

a. Choose one or two places in the problem-solving sequence to invite student responses. Have these choices in mind before you begin scaffolding instruction. (Examples of choices are shown in red.)

- Introduce the story problem.
  - "Ok, here we have another story problem." I know that I first need to read the story problem. By reading it, I will be able to find the important information that will help me solve the story problem. I know that reading it aloud can help me really hear the words as I read it.



- Read the story problem aloud and then have students read it with you.
  - "I'm going to read the story problem aloud first. Then I'd like you to read it with me the second time." (Read the story problem aloud, pointing to the words as you read them.)
  - Now, I'd like for you to read the story problem with me. (Read the story problem aloud with your students.)
- Teach finding the important information in the story problem.
  - Link process of finding important information of addition, subtraction, and multiplication story problems to division story problems. - "Now, that I've read the story problem and you helped me, I know from solving other story problems that involved addition, subtraction, and multiplication, that there is important information in the story problem that will help me find the solution."
  - Model finding what you are solving for. - "The first important information that I need to find in what I'm solving for. What do I look for when I want to find what I am solving for? (Elicit the response, "the question/question mark.") Yes, I look for the question. What do I do when I find the question? (Elicit the response, "underline the question and circle the question mark.") Yes, first I circle the question mark, because it tells me that this statement is a question and then I underline the question. (Circle the question mark and then underline the question.)"
  - Model finding the number phrases that represent the "dividend/total" and the "divisor." - "Now, I need to find the number phrases that represent my dividend, or total, and my divisor. (Point to the word cards that represent "dividend" and "divisor.") I know one strategy that can help me find the number phrases is to read each sentence and ask myself the question, "Is there a number phrase in this sentence?" What question do I ask, after I read each sentence? (Elicit the response, "is there a number phrase in this sentence?") Good. I'll do that now. (Model this process to find the number phrases that represent the dividend and the divisor. Emphasize the relationship between the number phrase, whether it is the dividend or the divisor and why it is the dividend or the divisor.)"
  - Model deciding if all the important information is identified. - "When I have finished finding the important information, it is helpful to check back and be sure I have found all of the important information. I have looked for two important things. (Have written on the chalkboard or dry-erase board "question" and "number phrases.") What two important things do I look for? (Point to the phrases on the board and elicit the response, "question" and "number phrases.") Good. First, I looked to find what I am solving for. I did this by finding the question. (Point to the circled question mark and the underlined question, then write a check beside the word "question" written on the

board.) The second important information I looked for were the number phrases. I read each sentence and asked myself the question, "is there a number phrase in this sentence?" I found two number phrases. (Point to the two number phrases that you circled and then write a check beside "number phrases" written on the board.) **Have we found all of the important information? (Point to the checked off phrases and elicit the response, "yes.")**"

- Model the division process by "dividing" the objects (separating them into equal groups by dealing objects evenly among small paper plates). Explicitly relate this action to the story context and relevant important information. Re-emphasize the language "dividend" and "divisor."
  - "Now that I know all the important information, I can use my concrete materials 'act out the story' and then to solve the problem. " I'll do this by reading each sentence and 'acting' it out with my concrete objects. (Read each sentence and "act" it out with the concrete objects, thinking aloud what you are doing and why. Represent the dividend with counting objects and the divisor with containers such as paper plates. Deal the counting objects one-by-one among the plates. Use visual cueing by pointing to each number phrases as you represent it.)"
- Model finding the solution to the story context with the concrete objects.
  - Review what you are solving for. - " Now that I've 'acted out' the story, its time to solve it. Hmm, how do I know what I'm solving for? (Elicit the response, "read the question.") Yes, the question tells me what I am solving for. The question asks....."
  - Find how many objects are on each plate. - "Ok, in order to solve this, I know I can count the number of objects on each plate. I have \_\_\_\_ counting chips on each plate." (Count aloud the counting chips on each plate to emphasize that each plate has the same number of objects.)"
  - Find the remainder. "I have \_\_\_\_ counting chips on each plate, but I have some counting chips left over. Let's see, I have \_\_\_\_ counting chips left over. Hmm, when I have objects left over, I know I need to make sure there aren't enough objects to put one more on every plate. Let's see, I have \_\_\_\_ counting chips left. I have \_\_\_\_ plates. I have fewer counting chips than I do plates. Therefore, I know I don't have enough counting chips to put one more on every plate. **What do I call the left over counting chips? (Point to the word card with "remainder" written, and elicit the response, "the remainder.")** Yes we call the left over objects, the remainder."
  - Identify the solution. "I can find the solution to the story problem by counting the number of counting chips on one plate because in the story, .....(Relate the question to the concrete objects.) I have \_\_\_\_ counting chips on each plate. Therefore, each \_\_\_\_\_ gets \_\_\_\_ \_\_\_\_\_. I also have \_\_\_\_ counting chips left over."

Therefore, the solution to the story problem is that each \_\_\_\_\_ gets \_\_\_\_\_  
\_\_\_\_\_ and there are \_\_\_\_\_ left over."

b. Maintain a high level of teacher direction/support for another example if students demonstrate misunderstanding/non-understanding; move to a medium level of teacher direction/support if students respond appropriately to the selected questions/prompts.

## 2.) Scaffold Using a Medium Level of Teacher Direction/Support

a. Choose several more places in the problem-solving sequence to invite student responses. Have these choices in mind before you begin scaffolding instruction.

- Introduce the story problem.
  - "Ok, here we have another story problem." I know that I first need to read the story problem. By reading it, I will be able to find the important information that will help me solve the story problem. I know that reading it aloud can help me really hear the words as I read it."
  
- Read the story problem aloud and then have students read it with you.
  - "What is the first thing I need to do when I have a story problem? (Elicit the response, "read it.") Yes. I'll going to read the story problem aloud first. Then I'd like you to read it with me the second time. (Read the story problem aloud, pointing to the words as you read them.)"
  - "Now, I'd like for you to read the story problem with me. (Read the story problem aloud with your students.)"
  
- Teach finding the important information in the story problem.
  - Link process of finding important information of addition, subtraction, and multiplication story problems to division story problems. - "Now, that I've read the story problem and you helped me, I know from solving other story problems that involved addition, subtraction, and multiplication, that there is important information in the story problem that will help me find the solution."
  - Model finding what you are solving for. - "The first important information that I need to find in what I'm solving for. What do I look for when I want to find what I am solving for? (Elicit the response, "the question/question mark.") Yes, I look for the question. What do I do when I find the question? (Elicit the response, "underline the question and circle the question mark.) Yes, first I circle the question mark, because it tells me that this statement is a question and then I underline the question. (Circle the question mark and then underline the question.)"

- o Model finding the number phrases that represent the “dividend/total” and the “divisor.” – “Now, I need to find the number phrases that represent my dividend, or total, and my divisor. (Point to the word cards that represent “dividend” and “divisor.”) **What strategy can I use to find the number phrases? (Elicit the response, “read each sentence and ask, “is there a number phrase in this sentence?”)** Good. I’ll do that now. (Model this process to find the number phrases that represent the dividend and the divisor. Emphasize the relationship between the number phrases, whether it is the dividend or the divisor and why it is the dividend or the divisor.)”
- o Model deciding if all the important information is identified. – “When I have finished finding the important information, it is helpful to check back and be sure I have found all of the important information. I have looked for two important things. (Have written on the chalkboard or dry-erase board “question” and “number phrases.”) **What two important things do I look for? (Point to the phrases on the board and elicit the response, “question” and “number phrases.”)** Good. First, I looked to find what I am solving for. **How did I find what I was solving for? (Elicit the response, “you looked for the question, circled the question mark and underlined the sentence.”)** **How did I find the number phrases? (Elicit the response, “you read each sentence and asked, ‘is there a number phrase in this sentence?’”)** Yes. **How many number phrases did I find? (Elicit the response, “two.”)** Great. I found two number phrases. (Point to the two number phrases that you circled and then write a check beside “number phrases” written on the board.) **Have we found all of the important information? (Point to the checked off phrases and elicit the response, “yes.”)**”
- Model the division process by “dividing” the objects (separating them into equal groups by dealing objects evenly among small paper plates). Explicitly relate this action to the story context and relevant important information. Re-emphasize the language “dividend” and “divisor.”
  - o “Now that I know all the important information, I can use my concrete materials to ‘act out the story’ and then to solve the problem. “ I’ll do this by reading each sentence and ‘acting’ it out with my concrete objects. (Read each sentence and “act” it out with the concrete objects, thinking aloud what you are doing and why.) **(Prompt students to think how to represent the “dividend.”)** **How do I represent the total? (Elicit the response, “count out that number of counting chips.”)** **What do we call the total? (Point to the “dividend” word card and elicit the response, “dividend.”)** (Represent the divisor with containers such as paper plates. Deal the counting objects one-by-one among the plates. Use visual cueing by pointing to the number phrase that represents the “divisor” as you represent it.)”
- Model finding the solution to the story context with the concrete objects.

- o Review what you are solving for. - " Now that I've 'acted out' the story, its time to solve it. **Hmm, how do I know what I'm solving for? (Elicit the response, "read the question.")** Yes, the question tells me what I am solving for. **What does the question ask? (Point to the question and elicit the appropriate response.)"**
- o Find how many objects are on each plate. - "Ok, in order to solve this, I know I can count the number of objects on each plate. I have \_\_\_\_ counting chips on each plate." (Count aloud the counting chips on each plate to emphasize that each plate has the same number of objects.)"
- o Find the remainder. - "I have \_\_\_\_ counting chips on each plate, but I have some counting chips left over. **When I have objects left over, what do I need to do? (Elicit the response, "count them to see if there are enough to put one more on every plate.")** Good. This is important to do, because, I want to divide as many counting chips as I can as long as each plate gets the same number. I know I don't have enough counting chips to put one more on every plate. **What do I call the left over counting chips? (Point to the word card with "remainder" written, and elicit the response, "the remainder.")** Yes we call the left over objects, the remainder."
- o Identify the solution. - "I can find the solution to the story problem by counting the number of counting chips on one plate because in the story, .....(Relate the question to the concrete objects.) I have \_\_\_\_ counting chips on each plate. Therefore, each \_\_\_\_\_ gets \_\_\_\_ \_\_\_\_\_. I also have \_\_\_\_ counting chips left over. Therefore, the solution to the story problem is that each \_\_\_\_\_ gets \_\_\_\_\_ and there are \_\_\_\_ \_\_\_\_\_ left over." **How many does each \_\_\_\_\_ get? (Elicit the appropriate response.) How many are left over? (Elicit the appropriate response.)** The name we use for the solution to a division problem is "quotient." (Point to the "quotient" word card.) Therefore, the solution, or quotient (Point to the "quotient" word card.) for this division problem is....."

b. Maintain a medium level of teacher direction/support for another example if students demonstrate misunderstanding/non-understanding; move to a low level of teacher direction/support if students respond appropriately to the selected questions/prompts.

### 3.) Scaffold Using a Low Level of Teacher Direction/Support

a. When students demonstrate increased competence, do not model the process. Ask students questions and encourage them to provide all responses. Direct students to replicate the process at their desks as you work together.

- Introduce the story problem.

- "Ok, here we have another story problem." What do we need to do? (Elicit the response, "read it.") Yes, we need to read it. Why will reading the story problem help us? (Elicit the response, "reading it will help us to find the important information that will help solve the story problem.") Great! Should we read it silently or out loud? (Elicit the response, "out loud.") Yes. Why should we read it out loud? (Elicit the response, "because it helps us hear the words as we read them.") Boy, you all are really sharp."
- Read the story problem aloud and then have students read it with you.
  - "What is the first thing I need to do when I have a story problem? (Elicit the response, "read it.") Yes. I'll going to read the story problem aloud first. Then I'd like you to read it with me the second time. " (Read the story problem aloud, pointing to the words as you read them.)"
  - "Now, I'd like for you to read the story problem with me. (Read the story problem aloud with your students.)"
- Teach finding the important information in the story problem.
  - "What should we look for in the story problem after we've read it? (Elicit the response, "the important information.") Good. What will "finding the important information" help us to do? (Elicit the response, "find the solution.") Yes, the important information will help us solve the story problem."
  - Model finding what you are solving for. – "What do I look for when I want to find what I am solving for? (Elicit the response, "the question/question mark.") Yes, I look for the question. What do I do when I find the question? (Elicit the response, "underline the question and circle the question mark.) Yes, first I circle the question mark, because it tells me that this statement is a question and then I underline the question. (Circle the question mark and then underline the question.)"
  - Model finding the number phrases that represent the "dividend/total" and the "divisor." – "After we've found what we are solving for, what important information do we look for next? (Elicit the response, "the number phrases.") Yes. What two names do we use for the number phrases in a division story problem? (Elicit the response, "the "dividend" and "divisor." \*Point to the word cards if needed.) What strategy can I use to find the number phrases? (Elicit the response, "read each sentence and ask, "is there a number phrase in this sentence?") Good. Let's do that now. (You and the students read through the story problem using this strategy to find the number phrases.)"
  - Model deciding if all the important information is identified. – "When we've finished finding the important information, what do we need to check for? (Elicit the response, "to be sure we've found all of the important information.") That's correct. What two important things do I look for? (Point to the phrases on the board and elicit the response, "question" and "number phrases.") Good. First, I looked to find what I am

solving for. How did I find what I was solving for? (Elicit the response, "you looked for the question, circled the question mark and underlined the sentence.") How did I find the number phrases? (Elicit the response, "you read each sentence and asked, 'is there a number phrase in this sentence?'" ) Yes. How many number phrases did I find? (Elicit the response, "two.") Great. I found two number phrases. (Point to the two number phrases that you circled and then write a check beside "number phrases" written on the board.) Have we found all of the important information? (Point to the checked off phrases and elicit the response, "yes.")"

- Model the division process by "dividing" the objects (separating them into equal groups by dealing objects evenly among small paper plates). Explicitly relate this action to the story context and relevant important information. Re-emphasize the language "dividend" and "divisor."
  - "Now that I know all the important information, I can use my concrete materials to 'act out the story' and then to solve the problem. "How should we 'act the story out'? (Elicit the response, "by reading each sentence and 'acting' it out with our chips.") Right. (Read each sentence aloud with students and "act" it out with the concrete objects.) (Prompt students to think how to represent the "dividend.") How do I represent the total? (Elicit the response, "count out that number of counting chips.") Great! (Ask students to hold up a chip to verify they recognize their counting chips.) What do we call the total? (Elicit the response, "dividend." \*Only point to the word card if additional cueing is needed.) Now, how do I represent the other number phrase? (Elicit the response, "with paper plates." ) Good. (Ask students to hold up a paper plate to verify they recognize their paper plates.)What do we call the number of paper plates? (Elicit the response, "the divisor." \*Only point to the word card if additional cueing is needed.") Great! Now what do we do? (Elicit the response, "deal the chips one-by-one among the plates.") Yes, we need to deal the counting chips one-by-one on each plate. Let's all do that now. (Ensure students deal their chips among the plates appropriately.)"
- Model finding the solution to the story context with the concrete objects.
  - Review what you are solving for. - " Now that I've 'acted out' the story, its time to solve it. How do I know what I'm solving for? (Elicit the response, "read the question.") Yes, the question tells me what I am solving for. What does the question ask? (Point to the question and elicit the appropriate response.)"
  - Find how many objects are on each plate. - "Ok, in order to solve this, what do we need to do? (Elicit the response, "count the number of chips on each plate.") Yes. Let's all do that now. (Encourage students to count the number of chips on their plates and ask for responses from several students.)"

- o Find the remainder. - "We have \_\_\_\_ counting chips on each plate, but we have some counting chips left over. When I have objects left over, what do I need to do? (Elicit the response, "count them to see if there are enough to put one more on every plate.") Good. Why is this important to do? (Elicit the response, "because we know for sure we've divided all the chips we can so each plate has the same number.") Yes. Are there enough chips to put one more chip on every plate? (Elicit the response, "no.") How many chips are there? (Elicit the appropriate response.) How many plates are there? (Elicit the appropriate response.) Good. What do I call the left over counting chips? (Point to the word card with "remainder" written, and elicit the response, "the remainder.") Yes we call the left over objects, the remainder."
- o Identify the solution. - "How do we find the solution to the story problem? (Elicit the response, "by counting the number of counting chips on one plate.") Excellent! Everybody count the number of chips on one plate. How many chips are there? (Elicit the appropriate response. Are their chips left over? (Elicit the response, "yes.") Right! How many chips are left over? (Elicit the appropriate response.) What is the solution to the story problem? (Elicit the appropriate response.) How many does each \_\_\_\_\_ get? (Elicit the appropriate response.) How many are left over? (Elicit the appropriate response.) What name do we use for the solution of a division problem? (Elicit the response, "quotient." \*Only point to the word card if additional cueing is needed.") What is the solution, or quotient for this division story problem? (Elicit the appropriate response, including the number of chips on each plate and the chips left over.)"

b. When you are confident students understand, ask individual students to direct the problem solving process or have the class direct you: Students ask questions and you and the students respond/perform the skill.

***Instructional Phase 2: Facilitate Acquisition to Mastery – Student Practice***

\*The student practice strategies described below can be used for both skills taught during initial acquisition through Teacher Directed Instruction. A detailed description for providing practice for one of the skills is provided below: Explicitly relate the place value of digits in one, two, and three digit numbers to where concrete materials are grouped on the place value mat.

***1. Receptive/Recognition Level***



*Purpose:* to provide students with many opportunities to determine which examples provided reflect the appropriate remainder.

**Learning Objective 4: Divide with remainders using concrete materials in measurement/"separating into equal groups" and partitive/"sharing" situations.**

Structured Language Experiences

*Materials:*

Teacher -

- concrete examples of solutions to various division equations with and without remainders.
- "choice" cards (three for each concrete example) that represent possible solutions to each concrete example. One card includes the appropriate solution. Appropriate language is used to represent the solutions (e.g. "four groups of five with two left over" would represent the solution to " $22 \div 5 = \underline{\quad}$ ").
- response sheet with correct solutions for answer key.

Students -

- response sheet numbered according to number of examples provided.
- pencil for writing

\*Appropriate accommodations for students with significant writing problems are to have them tape record their responses or have a letter written at the top of each "choice" card. Students can write the letter of the card they choose instead of writing the phrase.

\*An appropriate accommodation for students with reading difficulties is to pair them with a classmate who has the ability to read the language "choice" cards.

*Description:*

*Activity:*

Students work at a center where there are laid out a variety of concrete examples showing solutions to division equations (For example, the concrete solution to the division equation, " $16 \div 5 = \underline{\quad}$ " would be three groups of five counting objects and one counting object left over. \*It is important to group the counting objects in a distinct fashion so that the "remainder" can be clearly identified.). Above each concrete example are three cards with possible solutions written on them (For example, cards might read, "three groups of five with two left over," "three groups of five with one left over," "three groups of five with zero left over.". One card is the correct solution (with remainder). Students select which solution is appropriate and writes it down beside the appropriate number on their response sheet.

*"Structured Language Experience" Steps:*

- 1.) Review directions for completing structured language experiences/peer tutoring activity and relevant classroom rules.
- 2.) Model how to perform the skill(s) within the context of the activity *before* students begin the activity. Model both how to decide an appropriate choice and model how to write the solution on the response sheet.
- 3.) Provide time for student questions.
- 4.) Signal students to begin.
- 5.) Monitor students as they work. Provide positive reinforcement for both “trying hard,” responding appropriately, and for students using appropriate behavior. Also provide corrective feedback and modeling as needed.

## **2. Expressive Level**

*Purpose:* to provide students with multiple opportunities to solve division equations involving solutions “with remainders” and to describe the meaning of their concrete representations.

**Learning Objective 4: Divide with remainders using concrete materials in measurement/“separating into equal groups” and partitive/“sharing” situations.**

Structured Language Experiences/Structured Peer Tutoring

*Materials:*

Teacher –

- develop learning sheets – Each learning sheet includes division equations involving solutions “with remainders.” With each equation, the following questions structure student responding: How many total chips (or other appropriate concrete object)? How many plates (or other appropriate container)? How many chips on one plate? How many chips are left over?
- appropriate number of counting/discrete concrete objects and containers (e.g. counting chips, unifix cubes and paper plates.)
- Master answer key for learning sheet.

Students -

- learning sheets
- appropriate counting/discrete concrete objects and containers
- an answer key

*Description:*

*Activity:*

Students work in pairs, responding to the learning sheet. The student practice period is separated into two equal time periods. The coach presents the division equation by pointing to it and saying the equation aloud (e.g. points to the equation,  $8 \div 3 = \underline{\quad}$ , and says, "eight divided by three."). Then the coach asks each question that follows. The player responds to each question using the appropriate concrete objects and then saying the answer. The coach writes the answer in the appropriate space, checks the answer key and provides appropriate feedback (e.g. positive verbal reinforcement for accurate responses and corrective feedback for inaccurate responses.) For inaccurate responses, the coach provides feedback and the player attempts the question a second time. The first response is crossed out and the second response is recorded. The coach provides appropriate feedback as appropriate. The teacher signals students to "switch roles" at the appropriate time.

*"Structured Language Experiences/Structured Peer Tutoring" Steps:*

- 1.) Select pair groups and assign each pair a place to practice (try to match students of varying achievement levels if possible).
- 2.) Review directions for completing structured language experiences/peer tutoring activity and relevant classroom rules. Practice specific peer tutoring procedures as needed (see step #4).
- 3.) Model how to perform the skill(s) within the context of the activity *before* students begin the activity. Model both what the coach does (e.g. reads aloud the division equation) and how the player responds (e.g. using concrete materials).
- 4.) Divide the practice period into two equal segments of time. One student in each pair will be the player, or "talker/describer" and will solve the division equation using concrete materials and then describe their solution. The other student will be the coach, or "listener/evaluator" and will point to and then say aloud each problem. The coach will then write the response in the appropriate space, check the answer key, and provide feedback regarding the player's response (e.g. positive verbal reinforcement for accurate responses and corrective feedback for inaccurate responses.) For inaccurate responses, the coach provides feedback and the player attempts the question a second time. The first response is crossed out and the second response is recorded. The "listener/describer" will also tally corrects and in-corrects based on the player's responses.
- 5.) Provide time for student questions.
- 6.) Signal students to begin.
- 7.) Signal students when it is time to switch roles.
- 8.) Monitor students as they work in pairs. Provide positive reinforcement for both "trying hard," responding appropriately, and for students using appropriate tutoring behaviors. Also provide corrective feedback and modeling as needed.

***Instructional Phase 3: Evaluation of Student Learning/Performance (Initial Acquisition through Mastery/Maintenance)***

***1. Continuously Monitor & Chart Student Performance***

*Purpose:* to provide you with continuous data for evaluating student learning and whether your instruction is effective. It also provides students a visual way to “see” their learning.

*Materials:*

Teacher –

- appropriate prompts if they will be oral prompts
- appropriate visual cues when prompting orally

Student –

- appropriate response sheet/curriculum slice/probe
- graph/chart

*Description:*

*Steps for Conducting Continuous Monitoring and Charting of Student Performance:*

- 1.) Choose whether students should be evaluated at the receptive/recognition level or the expressive level.
- 2.) Choose an appropriate criteria to indicate mastery.
- 3.) Provide appropriate number of prompts in an appropriate format (receptive/recognition or expressive) so students can respond.
  - Based on the skill, your students’ learning characteristics, and your preference, the curriculum slice or probe could be written in nature (e.g. a sheet with appropriate prompts; index cards with appropriate prompts), or oral in nature with visual cues (e.g. say aloud the prompt and display it visually with concrete objects, drawings, written language, or numbers and symbols) or a combination of written curriculum slices/probes and oral prompts with visual cues.
- 4.) Distribute to students the curriculum slice/probe/response sheet/concrete materials.
- 5.) Give directions.
- 6.) Conduct evaluation.
- 7.) Count corrects and incorrects/mistakes (you and/or students can do this depending on the type of curriculum slice/probe used – see step #3).
- 8.) You and/or students plot their scores on a suitable graph/chart. A goal line that represents the proficiency (for concrete level skills, this should be %100 – 5 out of 5 corrects) should be visible on each students’ graph/chart).

9.) Discuss with children their progress as it relates to the goal line and their previous performance. Prompt them to self-evaluate.

10.) Evaluate whether student(s) is ready to move to the next level of understanding or has mastered the skill at the abstract level using the following guide:

*Concrete Level:* demonstrates %100 accuracy (given 3 to 5 response tasks) over three consecutive days.

*Representational Level:* demonstrates %100 accuracy of (given 8-10 response tasks) over two to three consecutive days.

*Abstract Level:* demonstrates near %100 accuracy for (two or fewer incorrects/mistakes) and a rate (# of corrects per minute) that will allow them to be successful when using that skill to solve real-life problems and when using the skill for higher level mathematics that require use of that skill.

11.) Determine whether you need to alter or modify your instruction based on student performance.

## **2. Additional Assessment Activity Appropriate For This Math Skill/Concept**

### Flexible Math Interview

*Purpose:* to evaluate student conceptual understanding of the division process, with and without remainders.

#### *Materials:*

Teacher -

- a small notepad to write notes regarding particular student's understanding as you "interview" them.
- pencil for writing

Students -

- appropriate concrete objects

#### *Description:*

As students are working independently or in pairs, ask them to describe their solutions and how they arrived at them. Encourage students to both use concrete materials to do this as well as "talk about" what they are doing with their concrete materials.

***Instructional Phase 4: Maintenance – Periodic Practice to Maintain Student Mastery of Skills***

*Purpose:* to provide students with opportunities to maintain their level of mastery of division using concrete materials.

**Problem of the Day**

*Materials:*

Teacher –

- a written prompt on the chalkboard, dry-erase board, or overhead projector (e.g. a division problem) or a concrete example of some part of the division process (e.g. solution to a division problem that includes a remainder).

Students –

- concrete materials if appropriate
- paper and pencil to record their responses if appropriate

*Description:*

Teacher presents a “problem of the day” that focuses on a particular skill or conceptual understanding of the division process (e.g. remainders). The problem can be written in nature or be represented with concrete materials. The “problem of the day” is displayed as students enter the room or as the period begins. Students are asked to “solve” the problem and provided necessary directions. After an appropriate amount of time, the teacher and the students “talk through” the problem and its solution. Students can individually describe how they approached the problem. Specific positive verbal reinforcement is provided by the teacher as well as specific feedback regarding misunderstandings students may have. Teacher notes students who seem to be having difficulty for the purpose of reviewing/re-modeling appropriate skills and concepts.