Instructional Plan

Concrete Level

Name of Math Skill/Concept: Comparing Fractions with Like and Unlike Denominators Using Concrete Materials

Prerequisite Skills Needed:

- Identify and name fractional parts using concrete materials.
- Identify numerator and denominator and understand what they represent.

Learning Objectives:

1.) Compare fractional parts with like denominators using concrete materials that represent the area/measurement model (e.g. circle pieces, fraction bars/strips, cuisenaire rods).

2.) Compare fractional parts with unlike denominators using concrete materials that represent the area/measurement model (e.g. circle pieces, fraction bars/strips, cuisenaire rods).

3.) Compare fractional parts with like denominators using concrete materials that represent the sets model (e.g. sets of counting chips, beans, unifix cubes, tickets).

4.) Compare fractional parts with unlike denominators using concrete materials that represent the sets model (e.g. sets of counting chips, beans, unifix cubes, tickets).

Important Ideas for Implementing This Teaching Plan:

1.) First introduce the concepts of comparing fractions with like and unlike denominators using concrete materials that represent "areas," such as pizza and pizza slices, circle pieces, fraction bars/strips, and Cuisenaire rods. Students who have learning problems will be able to better conceptualize the differences/sameness between fractional parts through the area model because they can visualize the parts in relationship to the whole and can also "feel" the value of each fraction by running their fingers around the area of the "whole" a fraction comprises.

2.) Teach and provide student practice for comparing fractions with like denominators before doing so for unlike denominators.

3.) Emphasize the importance of the relationship of the numerator to the denominator when comparing fractional parts, particularly when comparing fractions with like denominators. Students may rely on comparing the value of the numerators only, since this strategy will be a successful one with like denominators. However, this strategy will not be successful with unlike denominators. Such a strategy also prevents students from truly understanding what a fraction actually represents, which is the proportion of the "parts" to the "whole."

Instructional Phase 1: Initial Acquisition of Skill/Concept – Teacher Directed Instruction

Teach Skill/Concept within Authentic Context

Description: Two meaningful contexts are used to explicitly model these concepts/skills. For comparing fractions with like and unlike denominators (area model), the context is a pizza party. "Pizzas" and "pizza slices" made out of construction paper/tag board/cardboard are used. For comparing fractions with like and unlike denominators (sets model), the context is a visit to an amusement park where each student has a specified number of tickets to ride their favorite amusement rides.

Build Meaningful Student Connections

Purpose: to help students build meaningful connections between what they know about fractions/fractional parts and the process of comparing fractional parts.

*The following description is an example of how you might implement this instructional strategy for Learning Objective 1. A similar process can be used for the other learning objectives in this plan.

<u>Learning Objective 1:</u> Compare fractions with like denominators using concrete materials that represent the area model.

Materials:

Teacher –

- concrete materials representing the area model (e.g. circle pieces.)
- a written statement reflecting the learning objective that is visible to all students.
- A candy bar and utensil for cutting it into two pieces.
- A platform for displaying concrete objects that allows all students to see.

Description:

1.) **<u>L</u>** ink to students' prior knowledge of/experiences with fractions and with previous area model concrete materials used to develop understanding of fractions.

<u>For Example:</u>

(Display circle pieces that represent a "whole, one-half, one-fourth, and one-eighth; or, display a "whole" circle, a complete set of "one-half" pieces, a complete set of "one-fourth" pieces, and a complete set of "one-eighth" pieces. * When displaying the sets of "one-halves," "one-fourths," and "one-eighths," leave some space between the pieces) Who remembers what these materials represent? (Elicit appropriate responses.) Yes, these materials represent fractions/fractional parts. What does this one represent? (Point to the circle and elicit the response, "a whole.") Yes, this circle represents a whole. What does this piece represent? (Point to the "one-half" piece and elicit the response, "one-half.") Yes, this piece represents "one-half" because it is "one-half" of the circle. (Place the "one-half" piece on top of the circle to illustrate this point; or use complete sets of circle pieces and show that two "one-half" pieces make a whole circle.) *Continue this process for each circle piece or set of circle pieces.

2.) **I** dentify the skill students will learn: Compare two fractions to determine which fraction is greater, which one is less, or if they are equal.

For Example:

(Display a written statement of the learning objective that is visible to all students. – e.g. "Compare two fractions to determine which fraction is greater, which one is less, or if they are equal.") Today we are going to use materials like these (point to the displayed circle pieces) to compare two fractions to determine which fraction is greater, which fraction is less, or whether they are equal. (Point to the written learning objective as you say this.) What are we going to learn today? (Point to the written learning objective and elicit the response, "compare two fractions to determine which fraction is greater, which one is less, or if they are equal.") Yes, we are going to learn how to compare two fractions to determine which fraction is greater, which one is less, or if they are equal.

To get an idea of what we will learn to do with more complex fractions, we can compare two of the fraction pieces we have displayed here. Let's compare our "one-half" piece and our "one-fourth" piece. (Place the "one-half" piece and the "one-fourth" piece side-by-side.) Which fraction is greater than the other? (Elicit the response, "one-half.") Yes, the "one-half" piece is greater than the "one-fourth" piece. We can show that this is so by placing the "one-fourth" piece on top of the "one-half" piece. (Place the "one-fourth" piece on top of the "one-half" piece.) We can see that the "one-fourth" piece is less than the "one-half" piece, because part of the "one-half" piece is still showing. (Point to the portion of the "one-half" piece that is exposed.) This is the kind of thing we will be doing today, except we'll be learning how to compare many different fractions. We'll learn how to compare fractions that are even more complex than these.

3.) **P** rovide rationale/meaning for learning how to compare two fractions to determine which fraction is greater, which one is less, or if they are equal.

For Example:

Being able to compare fractions can be very helpful. Let me show you an example of why this is so. I have a candy bar here. (Show students a candy bar.) Now, say both Sharon and I both put in the same amount of money to buy this candy bar. It would be important that we both got an equal part of the candy bar when we cut it into two parts to eat. (Call the student to the front of the room.) Now say, I cut the candy bar like this. (Cut the candy bar so that one piece is about one-fourth in size and the other piece is the remaining three-fourths of the candy bar. This piece (hold up the "one-fourth" piece) is about one-fourth of the whole candy bar. This piece (hold up the "three-fourth" piece next to the "one-fourth" piece) is about three-fourths of the whole candy bar. Now, would it be fair for me to give Sharon the "one-fourth" piece? (Hand the student the smaller piece of the candy bar, and elicit the response, "no.") That's right, it wouldn't be fair. Why? (Elicit the response, "because she paid the same amount for the candy bar as you did so she should get an equal amount.") Excellent thinking! In this case, it is important for both Sharon and I to know that the candy bar needs to be cut into two "one-half" pieces so they are equal in size. Since we both paid the same amount for the candy bar, both of us should get an equal fraction of the whole candy bar. If we didn't know about fractions and how to compare them, we wouldn't be able to share the candy bar fairly.

Provide Explicit Teacher Modeling

Purpose: to provide students a clear teacher model of how to compare fractions with like and unlike denominators using concrete materials.

<u>Learning Objective 1:</u> Compare fractional parts with like denominators using concrete objects that represent the area model (e.g. circle pieces, fraction bars/strips, Cuisenaire rods).

Materials:

Teacher –

- story problems/contexts that represent situations comparing fractional parts with like denominators (greater, less, equal). *Story problems should be color coded (e.g. number phrase that represents denominator is color-coded blue and number phrases that represent the numerators are color-coded red.)

  <u>For Example:</u>

  Ms. Gray and several of her students had a pizza party to celebrate the wonderful job they did learning math by ordering both a cheese pizza and a pepperoni pizza that were both the same size! After the party was over, some pizza was left over. Each whole pizza the group ordered had a total of four pieces. There was one slice of cheese pizza left and two slices of pepperoni pizza left. What fractional part of pizza left over is greater, cheese or pepperoni?

- concrete materials that represent the area model including representations for the denominator as well as for the numerator –
  - Pizzas and pizza slices made from cardboard. Plain cardboard circles separated into the appropriate fractional parts should be used to represent the *denominator*. The numerator is represented by equal size pizza slices cut out of tag board or construction paper - e.g. four equal

pieces cut out for "fourths." *Cheese slices can be colored "yellow" and pepperoni slices can have pepperoni slices drawn on the "pepperoni slices.")

- circles/circle pieces - overhead pens or dry-erase markers can be used to draw lines that separate circles into appropriate fractional parts to represent the denominator
- fraction bars/strips
- cuisenaire rods

- Three "language cards:" one language card with "denominator" written; one language card with "numerator" written; and one language card with "like denominator" written. *Color code the language cards for "denominator" and "numerator" to match the color-coding used in the story problems.

Description:

A. Break down the skill of comparing fractional parts with like denominators using concrete objects that represent the area model.

1.) Introduce a story problem/context.
2.) Read the story problem aloud.
3.) Read the story problem with your students.
4.) Model finding what needs to be solved for in story problem.
5.) Model finding the important information in the story problem
6.) Represent the fractional parts with concrete objects.
7.) Compare the fractional parts (greater, less, equal).
8.) Solve the story problem.

B. Explicitly describe and model how to compare fractional parts with like denominators using concrete objects that represent the area model.

1.) Introduce a story problem/context.

- ➢ All students see it clearly
- ➢ Color-code denominator & numerator

For Example:

Here is a story problem that will help us learn how to use concrete materials to compare fractional parts.

Ms. Gray and several of her students had a pizza party to celebrate the wonderful job they did learning math by ordering both a cheese pizza and a pepperoni pizza that were both the same size! After the party was over, some pizza was left over. Each whole pizza the group ordered had a total of four pieces. There was *one slice of cheese pizza left and two slices of pepperoni pizza left. What fractional part of pizza left over is greater, cheese or pepperoni?*

2.) Read the story problem aloud.

- Point to words
- Read clearly
- Read at normal pace

For Example:

I'll read the story problem aloud first and then I'll ask you to read it aloud with me a second time. You can read along silently with me as I read the story problem aloud first. (Read the story problem aloud as you point to each word.)

3.) Encourage your students read the story problem with you a second time.

- Point to words
- Read clearly
- Read at normal pace

For Example:

Now, read the problem with me. (Read the story problem aloud a second time with your students. Continue to point to the words as you and your students read them together.).

4.) Model how to identify what you are solving for. (e.g. Look for question marks, circle the question mark and then underline the question.)

- Think aloud
- Skim story problem with finger
- Point to and circle question mark
- Point to, say, and underline question

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For Example:

Now that I've read the story problem, what do I need to do? (Elicit the response, "find the important information.") That's right, I need to find the important information. Hmm, I wonder what I need to find first. Oh, I remember. I first need to find what I am solving for. I know that one cue to help me find what I am solving for in a story problem is a question mark. Let me see if there are any question marks. (Move your finger over the story problem as if you are examining it to find any question marks.) I see a question mark at the end of the story problem. (Point to the question mark and circle it.) I know a question mark means the sentence it is with is a question. (Point to the beginning of the sentence and run your finger along the length of the sentence.) The question reads, "What fractional part of pizza left over is greater, cheese or pepperoni?" I can underline the question to help me remember what it is I need to solve for. So, what is it that I need to solve for in this story problem? (Elicit the response, "what fractional part of pizza left over is greater, cheese or pepperoni?") That's right. I need to determine what fractional part of pizza left over is greater, cheese or pepperoni? (Point to the question that asks for this information.)

5.) Model finding the important information in the story problem that will help you solve the story problem (e.g. Read each sentence and ask, "Is there a number phrase in this sentence?" – Circle the number phrases.).

- Read each sentence
- Ask, "Is there a number phrase/important information?"
- Circle important information

For Example:

I've found out what I'm solving for in the story problem. There is other important information I need to find as well. I know I've found important information in story problems before. What did I look for? Oh, I remember, looking for number phrases can help. I know a strategy that can help me find important information like number phrases. I can read each sentence in the story problem and ask myself, "Is there a number phrase in this sentence?" If it does, then I can circle the number phrase. I'll do that now. (Read each sentence and ask, "Is there a number phrase in this sentence?" Circle each number phrase.)

6.) Represent the fractional parts with concrete objects.

- Represent denominator
- Represent numerator
- Represent relationship of numerator to denominator
- Display "numerator" and "denominator" language cards

- Model how to represent the denominator.

For Example:

Now that I've identified what I need to solve for and the important information in the story problem, I can "act out" the story problem. It says the class had a pizza party and there is some pizza left over. What do I need to solve for? (Point to the questions and elicit the response, "whether there is more cheese or pepperoni pizza left and why.") Yes, I need to find out whether there is more cheese pizza left or pepperoni pizza left. I can use the important information I circled to do that. (Point to the circled number phrases.) My first number phrase is "total of four pieces." (Point to "total of four pieces.") What does this tell me? (Elicit the response, "how many pieces of pizza were in each whole pizza.") Yes, this number phrase tells me there were a total of four slices of pizza in each whole pizza. When we are dealing with fractions, what is the name we call the total number of parts in a whole? (Elicit the response, "denominator," and then display the language card with "denominator" written on it). Yes, the total number of parts of a whole is called the denominator.) How many whole pizzas did Ms. Gray and her students order? (Elicit the response, "two.") Yes, they ordered two pizzas (Point to the phrase in the story problem that says this.) I can represent the original pizzas with two cardboard circles that are each separated into four equal parts. (Display two cardboard circles that are clearly separated into four equal parts and tape them to the chalkboard/dry-erase board.) Each of these parts represents a slice of pizza Ms. Gray and her students ordered. (Point to the "parts" on the cardboard circles and count them aloud.) How many slices did each whole pizza have? (Elicit the response, "four.") Yes, there were four slices in each whole pizza. So, what is the denominator for each pizza? (Hold up the card with "denominator" written and elicit the response, "four.") Yes, "four" is the denominator because there were four total slices in each whole pizza. There is a special name we can use to describe situations where we have the same denominator. (Display the card with "like denominators," and point to the phrase.) We call them "like denominators.") What do we call denominators that are the same? (Point to the card with "like denominators" written, and elicit the response, "like denominators.") That's correct, we call denominators that are the same, "like denominators." (Point to the phrase "like denominator" on the card.)

- Model how to represent the numerator.

For Example:

Now that I have represented the denominator, I can find the next number phrase. The next number phrase is "one cheese pizza left." (Point to "one slice of cheese pizza left.") Hmm, how can I represent one slice of cheese pizza? Well, I can use this slice of cheese pizza I made from cardboard. (Show students the cardboard "slice" of cheese pizza and tape it on the chalkboard/dry-erase board.) Ok, the next number phrase is "three slices of pepperoni pizza." (Point to "two slices of pepperoni pizza left.") I can represent my two slices of pepperoni pizza with these pepperoni pizza slices I also made from cardboard. (Show students the cardboard "slices" of pepperoni pizza and tape them in a separate group next to the "slice" of cheese pizza.) Now, we have one slice of cheese pizza and two slices of pepperoni pizza. (Point to each group of slices as you say this.) The one cheese slice is a part of the original whole pizza and the two pepperoni slices represent parts of the original whole pepperoni pizza. What "fraction" name do we use for the parts of a whole? (Elicit the response, "numerator," and then display the card with "numerator" written.) Yes, the one cheese slice and the two pepperoni slices are the numerators in this fraction problem.

- Model representing the relationship of the numerator to the denominator (each "part" in fractional form).

For Example:

Now that we have represented all of the important information, what do I have to solve for? (Point to the questions and elicit the response, "what fractional part of pizza left is greater, cheese or pepperoni.") Yes, I need to determine what fractional part of pizza left is greater, cheese or pepperoni. At first I might think that I can simply count the number of cheese slices and pepperoni slices to solve the problem. However, since I'm working with fractions, I can't simply count the number of pizza slices. What other important information did I find in the story problem that I have to use in order to solve this fraction story problem? (Cue students by pointing to the phrase, "four slices in each whole pizza." and elicit the response, "how many slices of pizza are in each whole pizza/the denominator.") That's correct. I need to know how many total slices of pizza there were in the whole cheese and pepperoni pizzas that the group started with. How many slices were in each whole pizza? (Elicit the response, "four.") Yes, there were four slices in each whole pizza. I've represented the denominator in this story problem with cardboard circles that are separated into four equal parts. (Point to the two cardboard circles.) To solve this story problem, I have to compare the fractional parts of cheese and pepperoni pizza. How could I do this? (Elicit the response, "place the pizza slices on the cardboard circles.") That's right, I can place the remaining slices of cheese pizza on one of my cardboard circles that represent the denominator and then place the remaining pepperoni slices on the other cardboard circle. (Tape the cheese and pepperoni slices on top of the respective cardboard circles so that each slice "fits" into one of the "one-fourth" parts of the cardboard circle.)

7.) Model finding which fractional part is greater by comparing the concrete representations.

- Compare concrete representations
- Cue importance of the relationship of numerator and denominator

For Example:

Now that I have represented my left over cheese pizza slice and left over pepperoni slices in fraction form, I can compare them. What fractional part of cheese pizza do I have? (Point to the example of the "cheese pizza fraction" and elicit the response, "one-fourth.") Yes, there is "one-fourth" of a cheese pizza left. What fractional part of a pepperoni pizza do I have? (Point to the example of the "pepperoni pizza fraction" and elicit the response, "two-fourths.") Yes, I have there is "two-fourths" of a pepperoni pizza left. To solve the story problem, what do I need to determine? (Point to the question in the story problem and elicit the response, "what fractional part of pizza is greater, cheese or pepperoni?) Yes, I need to decide what fractional part of pizza is greater, the cheese fractional part (Point to the "cheese" fraction example.) or the pepperoni fractional part (Point to the "pepperoni" fraction example.) Well, which fractional part is greater, cheese or pepperoni? (Elicit the response, "pepperoni.") Yes, the fractional part of pepperoni pizza is greater. Why is the fractional part of pepperoni pizza greater than the fractional part of cheese pizza ? (Point to each example as you say this.) (Elicit the response, "because two-fourths is greater than "one-fourth;" because two parts out of four parts is more than one part out of four parts.") Great thinking! Two of four pepperoni slices represents a greater fractional part of a whole pizza than one of four cheese slices. I can see that this is so because when I look at the pepperoni example and compare it to the cheese example, I can see that more of a whole pepperoni pizza is left than of a whole cheese pizza. (Run your finger around the proportion of the whole pizza covered by the pizza slices for each example as you say this.)

- Cue students to why the *relationship of the numerator and denominator* in each fraction is important when comparing fractions.

For Example:

When we look at these pizzas, we can see *why* we have to consider the relationship of the numerator and the denominator of fractions that we compare. This relationship determines whether the fractional part is "greater than" or "less than" the other fractional part you are comparing it to. Let's take a look at our cheese pizza and pepperoni pizzas to see what I mean. The number of pepperoni slices left in our pepperoni pizza, representing the numerator, takes up more of the total area of the pepperoni pizza than the number of cheese slices in our cheese pizza. (Point to each pizza example as you say this, pointing first to the "left over" slices and running your finger around their area; then run your finger around the perimeter of the whole pizza.) The total areas of our pizzas are "made up" of the total number of slices, or our denominator. (Point to each of the "fourth" sections in each pizza.) So, by looking at both the slices that represent the numerator and the total number of slices in a whole pizza that represent the denominator. (First point to the "left over cheese slice" and then point to the two "left over" pepperoni slices.) Point to each of the "fourth" sections in each pizza), we can determine which fraction is greater.

8.) Model how to solve the story problem.

- Cue students to the question
- Prompt students to answer question
- Prompt students – "Why?"

For Example:

Now that we know which fraction of pizza is greater, we need to go back to our story problem and solve it. What do we need to solve for? (Point to the question in the story problem and elicit the response, "what fractional part of pizza left over is greater, cheese or pepperoni?") Yes, we need to answer the question, "What fractional part of pizza left over is greater, cheese or pepperoni?" What is the answer? (Elicit the response, "pepperoni.") That's right, "two fourths" of pepperoni pizza is a greater fractional part than the "one fourth" of cheese pizza. (Point to each example as you say this.) Why is "two fourths" greater than "one fourth?" (Elicit the response, "because more of the whole pepperoni pizza is left than cheese pizza.") That's right. The "two fourths" pepperoni pizza left is more of a whole pizza than the "one fourth" cheese pizza that is left.

9.) Repeat this process at least twice more each for comparisons of "greater than," "less than," and equal to" using different fractions with like denominators (e.g. 1/3 and 2/3; 3/6 and 5/6; 4/8 and 4/8). Include several different types of "area" manipulatives as students demonstrate understanding. It is suggested that when you introduce different types of area manipulatives, you start with those that are most like the ones you use to first model (e.g. circles/circle pieces would be an excellent choice if you first model the skill with "pizzas" and "pizza slices;"). As students demonstrate understanding, then move to examples using more different area models (e.g. fraction bars/strips; cuisenaire rods.)

***<u>Learning Objective 2:</u>* Compare fractional parts with unlike denominators using concrete materials that represent the area model.**

Materials:

Teacher –

- story problems/contexts that represent situations comparing fractional parts with unlike denominators (greater than, less than, equal to). *Story problems should be color coded (e.g. number phrase that represents denominator is color-coded blue and number phrases that represent the numerators are color-coded red.)

  <u>*For Example:*</u>

  Ms. Gray and several of her students had a pizza party to celebrate the wonderful job they did learning math by ordering a cheese pizza and a pepperoni pizza that were the same size! Although pizzas were the same size, the cheese pizza had a total of six slices and the pepperoni pizza had a total of eight slices. After the party was over, some pizza slices were left over. There were two slices of cheese pizza left and two slices of pepperoni pizza left. What fractional part of pizza left over is greater, cheese or pepperoni?
- concrete materials that represent the area model including representations for the denominator as well as for the numerator –
  - Pizzas and pizza slices made from cardboard. Plain cardboard circles separated into the appropriate fractional parts should be used to represent the *denominator*. The numerator is represented by equal size pizza slices cut out of tag board or construction paper - e.g. four equal pieces cut out for "fourths." *Cheese slices can be colored "yellow" and pepperoni slices can have pepperoni slices drawn on the "pepperoni slices.")
  - circles/circle pieces - overhead pens or dry-erase markers can be used to draw lines that separate circles into appropriate fractional parts to represent the denominator
  - fraction bars/strips
  - cuisenaire rods
- Three "language cards:" one language card with "denominator" written; one language card with "numerator" written; and one language card with "unlike denominator" written. *Color code the language cards for "denominator" and "numerator" to match the color-coding used in the story problems.

A. Break down the skill comparing fractional parts with unlike denominators using concrete materials that represent the area model.

1.) Introduce a story problem/context.

2.) Read the story problem aloud.

3.) Read the story problem with your students.

4.) Model finding what needs to be solved for in story problem.

5.) Model finding the important information in the story problem.

6.) Represent the fractional parts with concrete objects.

7.) Compare the fractional parts (greater, less, equal).

8.) Solve the story problem.

B. Explicitly Describe and Model how to compare fractional parts with unlike denominators using concrete materials that represent the area model.

1.) Introduce a story problem/context.

- All students see it clearly
- Color-code denominators and numerators

<u>For Example:</u>

Here is a story problem that will help us learn how to use concrete materials to compare fractional parts.

> Ms. Gray and several of her students had a pizza party to celebrate the wonderful job they did learning math by ordering a cheese pizza and a pepperoni pizza that were the same size! Although pizzas were the same size, the cheese pizza had a total of six slices and the pepperoni pizza had a total of eight slices. After the party was over, some pizza slices were left over. There were two slices of cheese pizza left and two slices of pepperoni pizza left. What fractional part of pizza left over is greater, cheese or pepperoni?

2. – 6.) Follow the same process, steps 2-6, described for Learning Objective 1, "comparing fractions with like denominators using concrete materials that represent the area model."

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7.) Follow the same process described for comparing fractions with like denominators (Learning Objective 1), including modeling how to represent the denominators, how to represent the numerator, representing each part in fractional form, and comparing the fractional parts represented with concrete objects.

1.) Represent denominator

2.) Introduce "unlike denominator" with language card

3.) Cue importance of relationship of numerator to denominator – cue area visually

4.) Prompt students – "Why?"

<u>Key Ideas:</u>

1. Be sure to emphasize that although the pizzas are the same size, the total number of slices for each pizza is different; therefore there are two different denominators. Represent the denominator, "six," with a plain cardboard circle divided into "sixths," and represent the denominator, "eight," with a plain cardboard circle

divided into “eighths.” Use the language card that reads, “unlike denominators” to introduce this term to students, just as you did with the term “like denominators” when teaching how to compare fractions with like denominators using concrete materials (Learning Objective 1) .

2. When comparing the fractional parts (e.g. two-sixths and two-eighths), explicitly emphasize that the number of slices of pizza/parts of a circle or fraction bar does not “automatically” determine which fractional part is greater or less.

- Cue students that it is *both* the number of parts (numerator) and the total number of parts (denominator) together that determine whether a fractional part is greater than, less than, or equal to another fractional part. Cue students visually by using your finger to show the “area” covered for each fractional part when using concrete representations like cardboard pizzas and pizza slices, fraction bars/strips, or Cuisenaire rods. If you are using overhead manipulatives such as circle piece, place the pieces that represent the numerator on top of a whole circle, then cue students by pointing out the “fractional” area that is colored differently from the rest of the whole circle.

-Ask students to describe why one fractional part is greater than, less than, or equal to another fractional part by coming up and “showing” as well as “saying” why it is so.

8.) Model how to solve the story problem.

- Cue students to the question
- Prompt students to answer question
- Prompt students – “Why?”

<u>For Example:</u>

Now that we know which fraction of pizza is greater, we need to go back to our story problem and solve it. What do we need to solve for? (Point to the question in the story problem and elicit the response, “what fractional part of pizza left over is greater, cheese or pepperoni?”) Yes, we need to answer the question, “What fractional part of pizza left over is greater, cheese or pepperoni?” What is the answer? (Elicit the response, “cheese.”) That’s right, “two sixths” of cheese pizza is a greater fractional part than the “two eighths” of cheese pizza. (Point to each example as you say this.) Why is “two sixths” greater than “two eighths?” (Elicit the response, “because more of the whole cheese pizza is left than the pepperoni pizza.”) That’s right. The “two sixths” cheese pizza left is more of a whole pizza than the “two eighths” pepperoni pizza that is left.

9.) Repeat this process at least twice more each for comparisons of “greater than,” “less than,” and equal to” using different fractions with unlike denominators (e.g. 2/3 and 4/8; 2/4 and 4/8; 2/5 and 3/6). Include several different types of “area” manipulatives as students demonstrate understanding. It is suggested that when you introduce different types of area manipulatives, you start with those that are most like the ones you use to first model (e.g. circles/circle pieces would be an excellent choice if you first model the skill with "pizzas" and "pizza slices;"). As students demonstrate understanding, then move to examples using more different area models (e.g. fraction bars/strips; cuisenaire rods.)

***<u>Learning Objective 3:</u>* Compare fractional parts with like denominators using concrete materials that represent the sets model (e.g. sets of counting chips, beans, unifix cubes, tickets.).**

Materials:

Teacher –

- story problems/contexts that represent situations comparing fractional parts with like denominators represented by sets (greater, less, equal). Story problems should be color coded (e.g. number phrase that represents denominator is color-coded blue and number phrases that represent the numerators are color-coded red.)

  <u>For Example:</u>

  Jenny and Carlos went with their parents to Kings Dominion Amusement Park this past weekend. Both Jenny and Carlos had a total of eight tickets to spend riding their favorite ride. Jenny used four of her tickets and Carlos used six of his tickets. Who used a greater fractional part of the total number of tickets given to them, Jenny or Carlos?

  - discrete concrete objects that can be grouped to sets model (e.g. "tickets" made from construction paper, counting chips, beans, unifix cubes, etc.)
  - paper plates to place each set of objects being compared.
  - pieces of string to encircle objects in sets that represent the numerator.
  - Three "language cards:" one language card with "denominator" written; one language card with "numerator" written; and one language card with "like denominator" written. Color code the language cards for "denominator" and "numerator" to match the color-coding used in the story problems.
  - markers for writing and highlighting.

A. Break down the skill of comparing fractional parts with like denominators using concrete materials that represent the sets model.

1.) Introduce a story problem/context.

2.) Read the story problem aloud.

3.) Read the story problem with your students.

4.) Model finding what needs to be solved for in story problem.

5.) Model finding the important information in the story problem.

6.) Represent the fractional parts with concrete objects.

7.) Compare the fractional parts (greater, less, equal).

8.) Solve the story problem.

B. Explicitly Describe and Model how to compare fractional parts with like denominators using concrete materials that represent the sets model.

1.) Introduce a story problem/context.

- All students see it clearly
- Color-code denominator and numerators

For Example:

Here is a story problem that will help us learn how to use concrete materials to compare fractional parts representing sets.

Jenny and Carlos went with their parents to Kings Dominion Amusement Park this past weekend. Both Jenny and Carlos had a total of eight tickets to spend riding their favorite ride. Jenny used four of her tickets and Carlos used six of his tickets. Who used a greater fractional part of the total number of tickets given to them, Jenny or Carlos?

2-5.) Follow the same process used in the previous descriptions for Learning Objectives 1 & 2 (comparing fractional parts with like and unlike denominators using the area model - reading the story problem, reading the story problem with your students, modeling how to find what you are solving for, and modeling how to find the important information in the story problem.)

6.) Represent the fractional parts with concrete objects.

- Represent denominator
- Represent numerator

- Model how to represent the denominator.

  - cue question to be solved

  - point to circled phrases

  - count out tickets on plates

- display “denominator” language card

For Example:

Now that I’ve identified what I need to solve for and the important information in the story problem, now I can “act out” the story problem. It says Jenny and Carlos went to the amusement park and were each given the same number of tickets to ride their favorite rides. Jenny and Carlos each used only a part of the total tickets they received. What do I need to solve for? (Point to the question and elicit the response, “who used a greater fractional part of the total number of tickets given to them, Jenny or Carlos?”) Yes, I need to find out whether Jenny or Carlos used a greater fractional part of the total number of tickets they were given. I can use the important information I circled to do that. (Point to the circled number phrases.) My first number phrase is “total of eight tickets.” (Point to “total of eight tickets.”) What does this tell me? (Elicit the response, “how many total tickets Jenny and Carlos got.”) Yes, this number phrase tells me that Jenny and Carlos each got a total of eight tickets. I can represent the eight tickets that both Jenny and Carlos received by counting out eight tickets on the two plates I have here. One plate will represent Jenny and one plate will represent Carlos. When we are dealing with fractions, what is the name we call the total number of parts? (Elicit the response, “denominator,” and then display the language card with “denominator” written on it). Yes, the total number of objects or parts is called the denominator. When we used concrete materials like circle pieces, the denominator represented the total number of equal parts the circle was separated into. For example, this circle is separated into a total of six equal parts. (Display a cardboard circle divided into “sixths” and count the number of equal parts.) When we have sets of objects, the total number of objects in the set is the denominator. The denominator represented here is the total number of tickets that Jenny and Carlos received. Both Jenny and Carlos each got eight tickets, so “eight” is our denominator.

- Model how to represent numerator (each “part”).
  - point to circled phrases
  - count tickets
  - circle tickets with string
  - display “numerator” language card

For Example:

Now that I ‘ve represented the denominator, I can find the next number phrase. The next number phrase is “Jenny used four of her tickets.” (Point to “Jenny used four of her tickets.”) Hmm, how can I represent the four tickets that Jenny used? Well, I can take this string and circle four tickets. (Circle four of the eight tickets represented on one plate, and then count the four tickets aloud.) The four tickets that I circled represent the four tickets that Jenny used. How many tickets did Jenny use? (Elicit the response, “four.”) Yes, Jenny used four tickets. How did I represent the four tickets Jenny used? (Elicit the response, “you circled four tickets on one plate with string.) Yes, the four tickets I circled with string represent the four tickets that Jenny used. Ok, the next number phrase is “Carlos used six of his tickets.” (Point to “Carlos used six of his tickets.") Just like I did with Jenny, I can represent the six tickets that Carlos used by circling them with string. (Circle six of the eight tickets represented on the other plate, and then count the six tickets aloud.) The six tickets that I circled represent the six tickets that Carlos used. How many tickets did Carlos use? (Elicit the response, "six.") Yes, Carlos used six tickets. How did I represent the six tickets Carlos used? (Elicit the response, "you circled six tickets on the other plate with string.) Yes, the six tickets I circled with string on this plate represent the six tickets that Carlos used. (Point to the plate representing Carlos.) Now, we have the four tickets that Jenny used represented here (Point to the plates with four tickets circled with string.) and we have the six tickets that Carlos used here (Point to the plate with six tickets circled with string.). The four tickets Jenny used is a part of the eight tickets she received and the six tickets Carlos used represent a part of the eight tickets he received. What "fraction" name do we use for the parts of a whole set? (Elicit the response, "numerator," and then display the card with "numerator" written.) Yes, the four tickets that Jenny used and the six tickets that Carlos used are the numerators in this fraction problem.

7.) Model how to compare the fractional parts and decide which is greater than, less than, or whether the fractional parts are equal.

- Prompt student thinking – What solve for?
- Emphasize importance of denominator
- Explicitly relate numerator to denominator
- Identify fractions
  - identify objects for numerator
  - identify objects for denominator
- Identify greater fraction

- Prompt students to think what needs to be solved for.

For Example:

Now that we have represented all of the important information, what do I have to solve for? (Point to the questions and elicit the response, " who used a greater fractional part of the total number of tickets given to them, Jenny or Carlos?") Yes, I need know to whether Jenny or Carlos used a greater fractional part of the tickets they were given.

- Emphasize the importance of the denominator when comparing fractions and model how to identify whether they are the same or different.

For Example:

At first I might think that I can simply count the number of tickets Jenny used and then compare that number to the number of tickets Carlos used. However, since I'm working with fractions, I can't simply count the number of tickets each person used. What other important information did I find in the story problem that I have to use in order to solve this fraction story problem? (Cue students by pointing to the phrase, "total of eight tickets." and elicit the response, "how many total tickets Jenny and Carlos got.") That's correct. I need to know how many total tickets Jenny and Carlos started with. Knowing the total number of tickets each person started with is very important if we are going to compare the fractional parts. This is important because a fraction represented by sets of objects includes both the number of objects circled/used, the numerator (Display the card with "numerator" written.), *and* the total number of objects in the set, the denominator (Display the card with "denominator" written). How many total tickets did each person start with? (Elicit the response, "eight.") Yes, both Jenny and Carlos each got eight tickets. I've represented the denominator in this story problem by counting out eight tickets on two different plates. (Point to the plates.) When we have denominators that are the same, what do we call them? (Display the card with "like denominators" written, and elicit the response, "like denominators.") That's correct, we call denominators that are the same, "like denominators." (Point to the phrase "like denominator" on the card.)

To solve this story problem, I have to compare the fractional part of the total number of tickets that Jenny used (Point to the circled tickets and then to all of the tickets for the plate representing Jenny) to the fractional part of the total number of tickets that Carlos used (Point to the circled tickets and then to all of the tickets for the plate representing Carlos). How could I compare the two fractional parts? (Elicit several responses.) Good ideas! Thanks for sharing what you think. Well, the first thing I need to do is to compare the denominators. What is the denominator for Jenny? (Elicit the response, "eight tickets.") Good. The denominator for Jenny is "eight." (Count aloud the eight tickets on the plate that represents Jenny.) What is the denominator for Carlos? (Elicit the response, "eight tickets.") Yes. The denominator for Jenny is also "eight." (Count aloud the eight tickets on the plate that represents Carlos.) Because the denominator, or the number of tickets Jenny and Carlos started with is the same, then we can compare the fractional parts by seeing who used a greater share of tickets in comparison to the eight tickets each got.

- Model how to determine the greater fractional part by relating the circled objects (numerator) to the total number of objects (denominator) in each set and determining which represents a greater share.

  <u>For Example:</u>

  How can we determine who used a greater share of their eight tickets? (Elicit the response, "count how many each used and see who used the most.") That's correct. I can count how many tickets Jenny used and compare that amount to how many tickets Carlos used. Who used the greater share of the eight tickets? (Elicit the response, "Carlos.") Yes, Carlos used a greater share of his eight tickets. How do you know this? (Elicit the response, "because he only has two of the eight tickets left while Jenny has four of eight tickets left." *It is important that you prompt students to consider not only the number of tickets each person used, but to relate that number to how many they had originally – pointing to the difference between the number they used and the total number is a good way to do this.) Great thinking! We always have to remember that since we are working with fractions, we have to always pay attention to the denominator. When we have sets of objects, like our two sets of eight tickets, then the denominator is the total number of objects in each set. If the denominator in the fractional parts we want to compare is the same, like they are in this example, then we can look to see how many objects in the original set were used. The set in which more objects are used, or circled (point to the two groups of tickets circled by string), will be the set that represents the greater fractional part. Which set represents the greater fractional part? (Elicit the response, "the set with six circled tickets/Carlos's set.") Yes, the set where six tickets were used represents the greater fractional part of the original eight tickets. Why does six out of eight tickets represent a greater fractional part than four out of eight tickets? (Elicit the response, "because Carlos used more of his eight tickets than Jenny.") Excellent thinking! Carlos and Jenny had the same total number of tickets. Because Carlos used more of his eight tickets than Jenny, then he used a greater share of his tickets than did Jenny.

- Prompt student thinking about the fractions each set represents and have your students say/identify those fractions. Explicitly identify the objects in each set that represent the numerator and the denominator in each fraction.

<u>For Example:</u>

What fraction does Jenny's set represent? (Point to the set with four circled tickets, and elicit the response, "four eighths.") Good. The four tickets that Jenny used, represented by the four circled tickets (Point to the four circled tickets) is the numerator and the total number of tickets (Point to and count aloud the eight total tickets) represent the denominator. The numerator is four and the denominator is eight, so the fraction represented by this set is "four eighths." (Based on your students' level of understanding, you may even prompt them to identify that "four eighths" is the same as "one-half.") What fraction does Carlos' set represent? (Point to the set with six circled tickets, and elicit the response, "six eighths.") Good. The six tickets that Carlos used, represented by the six circled tickets (Point to the six circled tickets) is the numerator and the total number of tickets represent the denominator. (Point to and count aloud the eight total tickets.) The numerator is "six" and the denominator is "eight," so the fraction represented by this set is "six eighths."

- Prompt students to identify which fraction is greater.

So, which fraction is greater, "four-eighths" or "six-eighths?" (Elicit the response, "six eighths.") Yes, "six-eighths" (Point to the set representing "six eighths.") is greater than "four eighths" (Point to the set representing "four eighths.")

8.) Model how to solve the story problem.

- Cue students to the question
- Prompt students to answer question

- Prompt students – "Why?"

For Example:

So, have we solved our story problem? (Point to the question in the story problem, and elicit the response, "yes.") Who used a greater fractional part of the total number of tickets given to them, Jenny or Carlos? (Elicit the response, "Carlos.") Yes, Carlos used a greater fractional part of his total tickets than did Jenny.

9.) Repeat steps 1-8 with at least two examples each for comparison of greater than, less than, and equal to. Use a variety of discrete concrete objects for sets.

Learning Objective 4: Compare fractional parts with unlike denominators using concrete materials that represent sets model (e.g. sets of counting chips, beans, unifix cubes, tickets).

Materials:

Teacher –

- story problems/contexts that represent situations comparing fractional parts represented by sets (greater, less, equal). *Story problems should be color coded (e.g. number phrase that represents denominator is color-coded blue and number phrases that represent the numerators are color-coded red.)

  For Example:

  Latrisa and David went with their parents to Kings Dominion Amusement Park this past weekend. Latrisa had a total of six tickets to spend riding her favorite rides. David had a total of eight tickets to spend riding his favorite ride. Latrisa used four of her tickets and David used four of his tickets. Who used a greater fractional part of the total number of tickets given to them, Latrisa or David?

  - discrete concrete objects that can be grouped to sets model (e.g. "tickets" made from construction paper, counting chips, beans, unifix cubes, etc.)
  - paper plates to place each set of objects being compared
  - pieces of string to encircle objects in sets that represent the numerator
  - Circles made from cardboard/tag board that are separated into halves, fourths, sixths, eighths, etc. (*Use a marker to draw the fractional parts on each cardboard circle.)
  - three "language cards:" one language card with "denominator" written; one language card with "numerator" written; and one language card with "unlike denominator" written. *Color code the language cards for "denominator" and "numerator" to match the color-coding used in the story problems
  - markers for writing and highlighting.

A. Break down the skill of comparing fractional parts with unlike denominators using concrete materials that represent the sets model.

1.) Introduce a story problem/context.

2.) Read the story problem aloud.

3.) Read the story problem with your students.

4.) Model finding what needs to be solved for in story problem.

5.) Model finding the important information in the story problem.

6.) Represent the fractional parts with concrete objects.

7.) Compare the fractional parts (greater, less, equal).

8.) Solve the story problem.

B. Explicitly Describe and Model how to compare fractional parts with unlike denominators using concrete materials.

1.) Introduce a story problem/context.

- All students see it clearly
- Color-code denominator & numerator

For Example:

Here is a story problem that will help us learn how to use sets of concrete objects like tickets, counting chips, an unifix cubes to compare fractional parts when the denominators are not the same.

Latrisa and David went with their parents to Kings Dominion Amusement Park this past weekend. Latrisa had a total of six tickets to spend riding her favorite rides. David had a total of eight tickets to spend riding his favorite ride. Latrisa used four of her tickets and David used four of his tickets. Who used a greater fractional part of the total number of tickets given to them, Latrisa or David?

2-6.) Follow the same process used in the previous description for Learning Objective 3 (comparing fractional parts with like denominators using set model) for reading the story problem, reading the story problem with your students, modeling how to find what you are solving for, modeling how to find the important information in the story problem, and representing the fractional parts with concrete objects.

7.) Model how to compare the fractional parts and decide which one is greater than, less than, or whether the fractional parts are equal.

- Prompt student thinking – "What solve for?"

- ➢ Emphasize importance of denominator
- ➢ Explicitly relate numerator to denominator
- ➢ Identify fraction
  - identify objects for numerator
  - identify objects for denominator
- ➢ Identify greater fraction
  - cue denominator with circles divided into fractional parts
  - place numerator (tickets) in fractional parts of circle

- Prompt students to think what needs to be solved for.

For Example:

Now that we have represented all of the important information, what do I have to solve for? (Point to the questions and elicit the response, " who used a greater fractional part of the total number of tickets given to them, Latrisa or David?") Yes, I need to know whether Latrisa or David used a greater fractional part of the tickets they were given.

- Emphasize the importance of the denominator when comparing fractions and model how to identify whether they are the same or different.

For Example:

At first I might think that I could simply count the number of tickets Latrisa used and then compare that number to the number of tickets David used. However, since I'm working with fractions, I can't simply count the number of tickets each person used. What other important information did I find in the story problem that I have to use in order to solve this fraction story problem? (Cue students by pointing to the two phrases, "total of six tickets" and "total of eight tickets," and elicit the response, "how many total tickets Latrisa and David got.") That's correct. I need to know how many total tickets Latrisa and David started with. Knowing the total number of tickets each person started with is very important if we are going to compare the fractional parts. This is important because a fraction represented by sets of objects includes both the number of objects circled/used, the numerator (Display the card with "numerator" written.), *and* the total number of objects in the set, the denominator (Display the card with "denominator" written). Did Latrisa and David start with the same number of tickets? (Elicit the response, "no.") That's correct, Latrisa started with six tickets (Point to the appropriate phrase in the story problem and then point to the plate with six total tickets.) and David started with eight tickets (Point to the appropriate phrase in the story problem and then point to the plate with eight total tickets.). So, is the denominator for Latrisa's set of tickets the same as the denominator for David's set of tickets? (Elicit the response, "no.") That's correct, the denominators are not the same. The denominator represented by Latrisa's set is "six." (Count aloud the six tickets on Latrisa's plate.) The denominator represented by David's set is "eight." (Count aloud the eight tickets on David's plate.) When we have denominators that are not the same, what do we call them? (Display the card with "unlike denominators" written, an elicit the response, "unlike denominators.") That's correct, we call denominators that are not the same, "unlike denominators." (Point to the phrase "unlike denominator" on the card.) What is the fraction represented by Latrisa's set? (Elicit the response, "four sixths.") Yes, Latrisa's set represents the fraction "four sixths." The four tickets she used is the numerator, "four" (Point to the four tickets circled with string.) and the six total tickets she had to begin with is the denominator "six" (Point to the six total tickets on Latrisa's plate.) What is the fraction represented by David's set? (Elicit the response, "four eighths.") Yes, David's set represents the fraction "four eighths." The four tickets he used is the numerator, "four" (Point to the four tickets circled with string.) and the eight total tickets he had to begin with is the denominator "eight" (Point to the six total tickets on Latrisa's plate.)

When comparing fractions, it is important to first determine whether the denominator in each fraction is the same or different/like or unlike. What is the first thing you need to do when you compare fractions? (Elicit the response, "see if the denominators are the same or different/are like or unlike denominators.") That's right, The first thing you need to do is to determine whether the denominators in the fractions you are comparing are the same or different/like or unlike denominators. Are the denominators in these two fractions, "four sixths" and "four eighths" the same or different/like of unlike? (Elicit the response, "different/unlike.") That's correct. The denominators in these two fractions, "four sixths" and "four eighths" are different/unlike. The denominator for Latrisa's set is "six." (Point to each of the six tickets on Latrisa's plate.) The denominator for David's set is "eight." (Point to each of the eight tickets on David's plate.)

- Prompt student thinking regarding why it is important to identify the denominator when comparing fractions and emphasize the importance of the relationship of the numerator (circled objects) to the denominator (total number of objects.).

<u>For Example:</u>

When we want to compare fractions, why is it important to identify whether the denominators are the same or different? (Elicit several responses.) Excellent ideas guys! Thanks for sharing what you think. When we compared fractions with "like denominators" using sets of objects, I told you that we could not simply count the number of circled objects to determine which fractional part was greater. I told you that when we are comparing fractions, it is very important to consider the share of objects circled in relation to the total number of objects. (Point to one set of circled objects and then run your finger the length of the total group of objects in the set.) This is even more important when we compare fractions with unlike denominators. When we compare fractions, can we simply compare the value of the numerator (number of circled objects) in each set? (Elicit the response, "no.") That's right, we cannot simply compare the value of the numerator (number of circled objects.) What do we need to look at? (Cue students by pointing to and moving your finger around the group of circled objects first and then point to the total group of objects, moving your finger around the total group of objects.) (Elicit the response, "the share of circled objects in relation to the total group of objects; both the circled objects and the total number of objects; the numerator and the denominator.") That's right we need to look at the share of circled objects in relation to the total number of objects. This is the fractional part. The fractional part is represented by both the numerator, in this case the group of circled objects (Point to the group of circled objects in one set.), and the denominator, in this case the total number of objects in each set (Point to total group of objects in one set.).

- Model how to determine the greater fractional part by relating the circled objects (numerator) to the total number of objects (denominator) in each set and by modeling how to determine which represents a greater share.

  - Prompt student thinking about how to determine who used a greater share of total tickets. Relate their rationale to previous experiences comparing sets representing fractions with "like denominators."

  <u>For Example:</u>

  How could we determine who used a greater share of their total number of tickets? (Elicit several responses.) Good thinking! Thanks for sharing you ideas. When we learned to compare sets representing fractions with "like denominators." we learned that we could compare the value of the numerator (circled objects). Who remembers why we could compare the value of the numerators? (Elicit the response, "because the denominator/total number of objects was the same; since the denominator for each set was the same, the set with more circled objects was a greater share of the total number of objects in each set.") That's right. Because both sets had the same number of total objects (denominator) the one with more circles objects represented a greater share/fraction of the total number of objects. But, since we are comparing unlike denominators in this problem, we can't do this. We can't do this because the total number of objects in each set is not the same.

    - Model how to compare sets representing fractions with unlike denominators by individually placing objects representing the numerator in "parts" of area model concrete objects that represent the same denominator (e.g. for a set representing "four sixths," individually place the four tickets that represent the numerator "four" in four adjacent "sixth" sections drawn on a cardboard circle.)

    <u>For Example:</u>

    Let me show you a way you can compare fractions represented by sets of objects that have unlike denominators. One way to do this is to use a circle that is separated into fractional parts. Each circle represents the denominator in each of our sets. Let me show you how to do this. I can use one circle to represent the denominator for Latrisa. What is the denominator for Latrisa? (Point to the total number of objects for Latrisa, and elicit the response, "six.") Yes, the denominator or total number of tickets Latrisa has is "six." How many fractional parts is this circle divided into? (Display the circle divided into "sixths," and elicit the response, "six.") Yes, there are six fractional parts to this circle. Another way to say this is that the circle is divided into "sixths." I can use another circle to represent the denominator for David. . What is the denominator for David? (Point to the total number of objects for David, and elicit the response, "eight.") Yes, the denominator or total number of tickets David has is "eight." How many fractional parts is this circle divided into? (Display the circle divided into "eighths," and elicit the response, "eight.") Yes, there are eight fractional parts to this circle. Another way to say this is that the circle is divided into "eighths." Since the denominator represented by the two circles is equal to the denominators in our two sets, I can use these circles to help me compare the fractional parts represented by the tickets Latrisa and David used. First, I'll place each of the four tickets Latrisa used in a separate "sixth" of the circle representing the denominator for Latrisa. (Place one ticket in adjacent "sixth" sections in the circle and count them aloud.) How many "sixths" have tickets? (Elicit the response, "four.") Yes, "four sixths" of the circle has tickets. (Outline the fractional part with a colored marker to emphasize the area represented by "four sixths.") Now let's do the same thing for the tickets David used. I'll place each of the four tickets that David used in a separate "eighth" of the circle representing the denominator for David. (Place one ticket in adjacent "eighth" sections in the circle and count them aloud.) How many "eighths" have tickets? (Elicit the response, "four.") Yes, "four eighths" of the circle has tickets. (Outline the fractional part with a colored marker to emphasize the area represented by "four eighths.") Which fractional part is greater, "four sixths" (Point to the circle and tickets representing "four sixths" and run your finger around the "four sixths" area.) or "four eighths?" (Pont to the circle and tickets representing "four eighths" and run your finger around the "four eighths" area.) (Elicit the response, "four sixths.") That's correct, "four sixths" is greater than "four eighths." How do you know this? (Elicit the response, "because more of the "four sixths" circle is covered than the "four eighths" circle.) That's correct, more of the circle representing "four sixths" is covered than the circle representing "four eighths." (Highlight each fractional part by running your finger around the areas they represent.)

8.) Model how to solve the story problem.

- Cue students to the question
- Prompt students to answer question
- Prompt student thinking – "Why?"

<u>For Example:</u>

Now that we know "four sixths" is greater than "four eighths," can we solve the story problem? (Elicit the response, "yes.") Yes, we can. What do we need to solve for? (Point to the question in the story problem and elicit the response, "who used a greater fractional part of the total number of tickets given to them, Latrisa or David?") Good. We need to determine who used a greater fractional part of the total number of tickets given them. So, who used a greater fractional part of the tickets given them? (Elicit the response, "Latrisa.") Yes, Latrisa used a greater fractional part than David. How do you know this? (Elicit the response, "because "four sixths" is greater than "four eighths; because four tickets out of six tickets is greater than four out of eight tickets; more of Latrisa's circle is covered than David's circle.") Great thinking! Because Latrisa used "four sixths" of her tickets and David used "four eighths" of his tickets, then Latrisa used a greater fractional part of her tickets. I know this because Latrisa's four tickets cover more of the "four sixths" circle than does David's four tickets that cover the "four eighths" circle. (Highlight the respective areas represented on the circles.)

9.) Repeat steps 1-8 with at least two examples each for comparisons of greater than, less than, and equal to. Use a variety of discrete concrete objects for sets.

Scaffold Instruction

Purpose: to provide students the opportunity to build their initial understanding of how to compare fractions with like denominators using concrete materials, and to provide you the opportunity to evaluate your students' level of understanding after your initial modeling of these skills.

*The steps for Scaffolding your instruction are the same for each concept you have explicitly modeled and with each Fraction Model you teach (Area and Sets). This teaching plan provides you a detailed example of scaffolding instruction for Learning Objective 1. A similar process can be used for the other learning objectives in this plan. You should Scaffold your instruction with each skill/concept you model..

<u>Learning Objective 1:</u> Compare fractions with like denominators using concrete materials that represent the area model.

Materials:

Teacher –

- visual display of story problem for comparing fractions with like denominators (area model); number phrases representing denominator and numerators are color-coded.
- concrete materials suitable for representing the denominator and numerators described in the story problem. (e.g. circles cut our of construction paper separated into equal parts that represent the denominator; circle pieces that represent halves, thirds, fourths, sixths, eighths and that correspond in area to the circles.)
- visual displays for the words "denominator," "numerator," and "like denominator. Color code "denominator" and "numerator" with the phrases in the story problem that represent these two concepts.

Also have available non-color coded language cards when such color-coding is no longer needed during the scaffolding process.

- markers/chalk for writing.

Students -

- copy of the story problem for comparing fractions with like denominators (area model) - *For Scaffolding Using a Low Level of Teacher Support.
- concrete materials suitable for representing the denominator and numerators described in the story problem. (e.g. circles cut our of construction paper separated into equal parts that represent the denominator; circle pieces that represent halves, thirds, fourths, sixths, eighths and that correspond in area to the circles.)
- pencils/pens/markers for writing.

Description:

1.) Scaffold Using a High Level of Teacher Direction/Support

a. Choose one or two places in the problem-solving sequence to invite student responses. Have these choices in mind before you begin scaffolding instruction. (Examples of choices are shown in red.)
Explicitly relate this action to the story context used during Explicit Teacher Modeling. Re-emphasize any new language introduced during Explicit Teacher Modeling (e.g. "like denominators").

- Introduce a story problem/context and read the story problem aloud with your students.
  - "Here's another story problem that involves comparing fractional parts. I'd like for you to read the story problem with me." (Read the story problem aloud with your students.)

- Model how to identify what you are solving for (e.g. Look for question marks, circle the question mark and then underline the question.)

  - "What do I look for when I want to find what I am solving for? (Elicit the response, "the question/question mark.") Yes, I look for the question. What do I do when I find the question? (Elicit the response, "underline the question and circle the question mark.) Yes, first I circle the question mark because it tells me that this statement is a question and then I underline the question." (Circle the question mark and then underline the question.)

- Model finding the important information in the story problem that will help you solve the story problem (e.g. Read each sentence and ask, "Is there a number phrase in this sentence?" – Circle the number phrases.).

    - Model finding the number phrases that represent the "denominator" and the "numerators." – "After we've found what we are solving for, what important information do we look for next? (Elicit the response, "the number phrases.") Yes. What strategy can I use to find the number phrases? (Elicit the response, "read each sentence and ask, "is there a number phrase in this sentence?") Good. Let's do that now." (You and the students read through the story problem using this strategy to find the number phrases.)

    - Model deciding if all the important information is identified. – " When we've finished finding the important information, what do we need to check for? (Elicit the response, "to be sure we've found all of the important information.") That's correct. What two important things do I look for? (Point to the phrases on the board and elicit the response, "question" and "number phrases.") Good. First, I looked to find what I am solving for. How did I find what I was solving for? (Elicit the response, "you looked for the question, circled the question mark and underlined the sentence.") How did I find the number phrases? (Elicit the response, "you read each sentence and asked, 'is there a number phrase in this sentence?'") Yes. How many number phrases did I find? (Elicit the response, "two.") Great. I found two number phrases. (Point to the two number phrases that you circled and then write a check beside "number phrases" written on the board.) Have we found all of the important information?" (Point to the checked off phrases and elicit the response, "yes.")

- Represent the fractional parts with concrete objects.

    - Model how to represent the denominator. – "Now that I know what I am solving for and have circled the important information, then I can "act" out my story problem with these concrete objects. How can I represent my first number phrase? Well, it says…… I can represent it with…… What is the name for the total number of parts in a fraction problem. Oh, I remember. It's called the denominator. (Display "denominator" language card.) So, there were a total of ____________ slices in both the cheese pizza and the pepperoni pizza. I can represent the total number of slices for each pizza with these ___________________. See, each circle/pizza has a total of ____________ pieces/slices. These _______________ pieces/slices represent my denominator. (Point to "denominator" language card. Hmm, are the denominators for each pizza the same? Well, yes they are. Each whole pizza had ____________ total pieces. I know there is a name for denominators that are the same. They are called "like denominators." (Display the "like denominators" language card.) What do we call denominators that are the same? (Elicit the response, "like denominators.") Yes, we call denominators that are the same, 'like denominators.'"

- Model how to represent the numerator. – "How can I represent my second number phrase? Well, it says…… I can represent it with…… What is the name for the number of parts of a whole in a fraction problem? Oh, I remember. It's called the numerator. (Display "numerator" language card.) So, there were_____________ slices of cheese pizza left over. (Count the number of pieces representing the cheese slices left.) What is the next number phrase? I can represent it with….. So, there were _____________ slices of pepperoni pizza left. . (Count the number of pieces representing the pepperoni slices left.) This number is also called a "numerator" since it represents parts or slices of a whole pizza. So, I have two numerators represented here. There are _____________ number of cheese slices left and ______________ number of pepperoni slices left."

- Model representing each "part" in fractional form. – "Now I need to represent the fractional parts the left over slices make. I know I can do this by placing the pieces left over on top of the whole circles that represent my denominator. (Model placing the pieces/slices that represent the fractional parts on top of the circles that represent the denominator.) Ok, I have _________ cheese slices/pieces out of a total of _________________ cheese slices/pieces. (Point to the pieces and then to each of the parts of the whole circle.) I also have ________________ pepperoni slices/pieces out of a total of __________________ pepperoni slices/pieces. Doing this will help me compare the fractional parts represented by the left over slices/pieces of cheese and pepperoni pizza."

- Model finding which fractional part is greater by comparing the concrete representations – "How can I determine which fractional part is greater? Well, I can look at how much area/space the left over slices in the both my cheese and pepperoni pizzas slices take up. I know the slices that take up the most area/space will be the fractional part that is greatest. To help me, I can run my finger around the area taken up by my cheese slices. (Run your finger around the area covered by "cheese" slices.) Ok, I have a good idea of how much space the cheese pizza slices take up. I can do the same thing with my pepperoni slices. (Run your finger around the area covered by "pepperoni" slices.) Now I have a good idea of how much space the pepperoni pizza slices take up. Based on "seeing" and "feeling" the area taken up by the cheese and pepperoni slices, I can tell that the fractional part represented by the ______________ slices is greater. ___________________ (name the fraction) is greater than __________________ (name the other fraction)."

- Model how to solve the story problem. – "Now that I know which fractional part is greatest, how can I solve the story problem? I need to find my question again. (Point to the question.) It says……. Well, I know the fractional part represented by my ______________ slices is greater than the fractional part represented by my ________________ slices. Therefore the ___________________(name fraction) of _______________ pizza left is greater than the ____________________ (name fraction) of ____________________ left."

b. Maintain a high level of teacher direction/support for another example if students demonstrate misunderstanding/non-understanding; move to a medium level of teacher direction/support if students respond appropriately to the selected questions/prompts.

2.) Scaffold Using a Medium Level of Teacher Direction/Support

a. Choose several more places in the problem-solving sequence to invite student responses. Have these choices in mind before you begin scaffolding instruction.

- Introduce a story problem/context and read the story problem aloud with your students.
  - "Here's another story problem that involves comparing fractional parts. I'd like for you to read the story problem with me." (Read the story problem aloud with your students.)

- Model how to identify what you are solving for (e.g. Look for question marks, circle the question mark and then underline the question.)

    - "What do I look for when I want to find what I am solving for? (Elicit the response, "the question/question mark.") Yes, I look for the question. What do I do when I find the question? (Elicit the response, "underline the question and circle the question mark.) Yes, first I circle the question mark, because it tells me that this statement is a question and then I underline the question." (Circle the question mark and then underline the question.)

- Model finding the important information in the story problem that will help you solve the story problem (e.g. Read each sentence and ask, "Is there a number phrase in this sentence?" – Circle the number phrases.).

  - Model finding the number phrases that represent the "denominator" and the "numerators." – "After we've found what we are solving for, what important information do we look for next? (Elicit the response, "the number phrases.") Yes. What two names do we use for the number phrases in a division story problem? (Elicit the response, "the "dividend" and "divisor." *Point to the word cards if needed.) What strategy can I use to find the number phrases? (Elicit the response, "read each sentence and ask, "is there a number phrase in this sentence?") Good. Let's do that now." (You and the students read through the story problem using this strategy to find the number phrases.)

- Model deciding if all the important information is identified. – "When we've finished finding the important information, what do we need to check for? (Elicit the response, "to be sure we've found all of the important information.") That's correct. What two important things do I look for? (Point to the phrases on the board and elicit the response, "question" and "number phrases.") Good. First, I looked to find what I am solving for. How did I find what I was solving for? (Elicit the response, "you looked for the question, circled the question mark and underlined the sentence.") How did I find the number phrases? (Elicit the response, "you read each sentence and asked, 'is there a number phrase in this sentence?'") Yes. How many number phrases did I find? (Elicit the response, "two.") Great. I found two number phrases. (Point to the two number phrases that you circled and then write a check beside "number phrases" written on the board.) Have we found all of the important information?" (Point to the checked off phrases and elicit the response, "yes.")

- Represent the fractional parts with concrete objects.

  - Model how to represent the denominator. – "Now that I know what I am solving for and have circled the important information, then I can "act" out my story problem with these concrete objects. What is the first number phrase? (Elicit the appropriate response.) How can I represent my first number phrase? (Elicit the response, "with a circle separated into ______ equal parts.) Excellent thinking! (Display the concrete materials that represent the denominator for each pizza/whole.) What is the name for the total number of parts in a fraction problem? (Display "denominator" language card and elicit the response, "denominator.") How many total slices/pieces are there in each pizza? (Point to the circle and the regions that represent the individual slices/pieces and elicit the appropriate response.) Are the denominators for each pizza the same? (Elicit the response, "yes.") What do we call denominators that are the same? (Display the "like denominators" language card and elicit the response, "like denominators.") Yes, we call denominators that are the same, 'like denominators.'"

  - Model how to represent the numerator. – "What is the second number phrase? (Elicit the appropriate response.) How can I represent my second number phrase? (Elicit the appropriate response.) Great thinking! (Display the concrete materials that represent the first numerator.) What is the name for the number of parts of a whole in a fraction problem (Display "numerator" language card and elicit the response, "numerator.") (*Repeat this process for the third number phrase.) How many cheese slices were left? (Elicit the appropriate response.) How many pepperoni slices were left? (Elicit the appropriate response.) So what is the numerator for cheese slices? (Elicit the appropriate response.) What is the numerator for pepperoni slices?" (Elicit the appropriate response.)

- Model representing each "part" in fractional form. – "Now I need to represent the fractional parts the left over slices make. I know I can do this by placing the pieces left over on top of the whole circles that represent my denominator. (Model placing the pieces/slices that represent the fractional parts on top of the circles that represent the denominator.) Ok, I have __________ cheese slices/pieces out of a total of __________________ cheese slices/pieces. (Point to the pieces and then to each of the parts of the whole circle.) I also have _________________ pepperoni slices/pieces out of a total of ___________________ pepperoni slices/pieces. Doing this will help me compare the fractional parts represented by the left over slices/pieces of cheese and pepperoni pizza."

  - Model finding which fractional part is greater by comparing the concrete representations. – "How can I determine which fractional part is greater? Well, I can look at how much area/space the left over slices in the both my cheese and pepperoni pizzas slices take up. I know the slices that take up the most area/space will be the fractional part that is greatest. To help me, I can run my finger around the area taken up by my cheese slices. (Run your finger around the area covered by "cheese" slices.) Ok, I have a good idea of how much space the cheese pizza slices take up. I can do the same thing with my pepperoni slices. (Run your finger around the area covered by "pepperoni" slices.) Now I have a good idea of how much space the pepperoni pizza slices take up. Based on "seeing" and "feeling" the area taken up by the cheese and pepperoni slices, I can tell that the fractional part represented by the ______________ slices is greater. ____________________ (name the fraction) is greater than ___________________ (name the other fraction)."

  - Model how to solve the story problem. – "Now that I know which fractional part is greatest, how can I solve the story problem? I need to find my question again. (Point to the question.) It says……. Well, I know the fractional part represented by my ______________ slices is greater than the fractional part represented by my ________________ slices. Therefore the ___________________(name fraction) of _______________ pizza left is greater than the _____________________ (name fraction) of ____________________ left."

b. Maintain a medium level of teacher direction/support for another example if students demonstrate misunderstanding/non-understanding; move to a low level of teacher direction/support if students respond appropriately to the selected questions/prompts.

3.) Scaffold Using a Low Level of Teacher Direction/Support

a. When students demonstrate increased competence, do not model the process. Ask students questions and encourage them to provide all responses. Direct students to replicate the process with appropriate concrete materials at their desks as you work together.

- Introduce a story problem/context and read the story problem aloud with your students.
  - "You have in front of you a copy of a story problem that involves comparing fractional parts. I'd like for you to read the story problem with me." (Read the story problem aloud with your students.)

- Model how to identify what you are solving for (e.g. Look for question marks, circle the question mark and then underline the question.)

    - "What do we look for when we want to find what we're solving for? (Elicit the response, "the question/question mark.") Yes, we look for the question. What do we do when we find the question? (Elicit the response, "underline the question and circle the question mark.) Yes, first we circle the question mark, because it tells me that this statement is a question and then we underline the question. Everyone circle the question mark and underline the question."
- Model finding the important information in the story problem that will help you solve the story problem (e.g. Read each sentence and ask, "Is there a number phrase in this sentence?" – Circle the number phrases.).

  - Model finding the number phrases that represent the "denominator" and the "numerators." – "After we've found what we are solving for, what important information do we look for next? (Elicit the response, "the number phrases.") Yes. What strategy can we use to find the number phrases? (Elicit the response, "read each sentence and ask, "is there a number phrase in this sentence?") Good. Do that on your own now. (Monitor students as they do this.) What is the first number phrase?" (Elicit the appropriate response. *Continue this process for the remaining number phrases.)

  - Model deciding if all the important information is identified. – "When we've finished finding the important information, what do we need to check for? (Elicit the response, "to be sure we've found all of the important information.") That's correct. What two important things do I look for? (Elicit the response, "question" and "number phrases.") Good. First, we looked to find what are solving for. How did we find what we are solving for? (Elicit the response, "we looked for the question, circled the question mark and underlined the sentence.") How did we find the number phrases? (Elicit the response, "we read each sentence and asked, 'is there a number phrase in this sentence?'") Yes. How many number phrases did I find? (Elicit the appropriate response.) Great. we found ______ number phrases. Have we found all of the important information? (Elicit the response, "yes.")"

- Represent the fractional parts with concrete objects.

    - Model how to represent the denominator. – "Now that we know what we're solving for and have circled the important information, then what do we do? (Elicit the response, "act" out our story problem with these concrete objects.) Yes. What is the first number phrase? (Elicit the appropriate response.) What is the name we use for total number of parts in a fraction problem? (Elicit the response, "denominator.") How can we represent my first number phrase? (Elicit the response, "with a circle separated into ______ equal parts.) Excellent thinking! Place the appropriate circle in front of you. (Monitor students as they do this and check to see that all students use the appropriate concrete material.) How many total slices/pieces are there in each pizza? (Elicit the appropriate response.) Are the denominators for each pizza the same? (Elicit the response, "yes.") What do we call denominators that are the same? (Elicit the response, "like denominators.") Yes, we call denominators that are the same, 'like denominators.'"

    - Model how to represent the numerator. – "What is the second number phrase? (Elicit the appropriate response.) What is the name for the number of parts of a whole in a fraction problem (Elicit the response, "numerator.") How can we represent the numerator described by the second number phrase? (Elicit the appropriate response.) Great thinking! (Display the concrete materials that represent the first numerator. (Repeat this process for the third number phrase.) How many cheese slices were left? (Elicit the appropriate response.) How many pepperoni slices were left? (Elicit the appropriate response.) So what is the numerator for cheese slices? (Elicit the appropriate response.) What is the numerator for pepperoni slices? (Elicit the appropriate response.)"

- Model representing each "part" in fractional form. – "Now that we have our numerators identified and we know what our denominator is, what do we need to do in order to represent the fractional parts represented by the left over cheese and pepperoni slices? (Elicit the response, "place the slices/pieces that represent the left of cheese and pepperoni pizza on top of the circles that represent the whole pizzas.") Excellent thinking! Everyone do that now. (Monitor students as they do this and check understanding.) How many cheese slices are there? (Elicit the appropriate response.) How many total cheese slices were there in the whole pizza? (Elicit the appropriate response.) How many pepperoni slices are there? (Elicit the appropriate response.) How many total pepperoni slices were there in the whole pepperoni pizza? (Elicit the appropriate response.) Why do we need to place our cheese and pepperoni slices on top of the whole pizzas? (Elicit the response, "to compare the fractional parts represented by the left over cheese slices and pepperoni slices.") Wonderful thinking!"

    - Model finding which fractional part is greater by comparing the concrete representations. – "How can we determine which fractional part is greater? (Elicit the response, "we can look at how much area/space the left over slices in the both our cheese and pepperoni pizzas slices take up) Yes. What else can you do? (Elicit the response, "run our finger around the area taken up by the cheese and pepperoni slices to feel the area.) Good thinking! Everyone do that now. (Monitor students and check for understanding.) Based on "seeing" and "feeling" the area taken up by your cheese and pepperoni slices, which fractional part is greatest, the fractional part represented by the cheese slices or the fractional part represented by the pepperoni slices? (Elicit the appropriate response.) What is the fraction name for the left over cheese slices? (Elicit the appropriate response.) What is the fraction name for the left over pepperoni slices? (Elicit the appropriate response.) Which fraction is greatest? (Elicit the appropriate response.)"

- Model how to solve the story problem. – "Now that we know which fractional part is greatest, how can we solve the story problem? (Elicit the response, "answer the question we underlined.") What is the question? (Elicit the appropriate response.) What is the answer? (Elicit the appropriate response.) Write the answer." (Monitor students and check for understanding. *Write the answer on the chalkboard/dry-erase board after students have had the opportunity to respond.)

b. When you are confident students understand, ask individual students to direct the problem solving process or have the class direct you: Students ask questions and you and the students respond/perform the skill.

Instructional Phase 2: Facilitate Acquisition to Mastery – Student Practice

1. Receptive/Recognition Level

Purpose: To provide students multiple opportunities to compare a variety of concrete representations of fractions (like or unlike denominators) and choose whether the first concrete representation is greater than, less than, or equal to the second concrete representation. Students also are provided multiple opportunities to describe why one fraction is greater than, less than, or equal to the other fraction.

***<u>Learning Objectives 1 - 4:</u>* Comparing fractions with like or unlike denominators using concrete materials (Area or Sets Model).**

Structured Peer Tutoring/Structured Language Experiences

Materials:

Teacher –

- "centers" where fractions are represented with the appropriate type of concrete material.
- Concrete fraction representations are aligned into pairs to be compared.
- language cards that "name" each concrete fraction representation are placed underneath each example.
- An "answer key" is prepared for each center that lists the fractional comparisons presented at each center (to be used for checking student response sheets.)

Students -

- a set of "response cards," one with "greater than," one with "less than," and one with "equal to."
- response sheets that are numbered according to the number of comparisons provided at the centers. (*response sheets can be coded by center so the teacher can match it with the appropriate answer key.)
- Pencil for writing.

Description:

Activity:

Students work in pairs at centers that have various concrete representations of fractions displayed (e.g. "two-sixths" is represented by two "sixth" circle pieces placed on a circle divided into six equal parts.) Concrete representations are displayed in pairs so that it is clear which ones are to be compared. Fractional names are written underneath each fractional representation (e.g. "two- sixths"). Each student has a set of cards that read "greater than," "less than," and "equal to" and a response sheet numbered based on the number of comparisons provided at the centers. Students take turns responding to examples. For each pair of fractions to be compared, one student ("player") decides whether the first fractional representation is greater than, less than, or equal to the second fractional representation. The student places the appropriate card between the two concrete representations. Then the student describes to their "peer" why they made the comparison they did. The other student ("coach") checks the "player's" response and provides appropriate feedback. If a pair has questions, the "coach" raises their hand for help from the teacher. The coach records the player's response ("greater than," less than," or "equal to") on the player's response sheet. At the conclusion of the activity, students turn in their response sheets to the teacher. The teacher monitors students as they work in pairs by circulating the room, answering questions, providing specific feedback, providing positive feedback, and modeling as needed.

Structured Language Experiences/Structured Peer Tutoring Steps:

1.) Select pair groups and assign each pair a place to practice (try to match students of varying achievement levels if possible).

2.) Review directions for completing structured language experiences/peer tutoring activity and relevant classroom rules. Practice specific peer tutoring procedures as needed (see step #4).

3.) Model how to perform the skill(s) within the context of the activity *before* students begin the activity. Model both what the coach does (e.g. checks player's response, provides appropriate feedback, writes response on player's response sheet) and how the player responds (e.g. using cards to identify "greater than," "less than," or "equal to;" describe why they chose the response they did.).

4.) Direct students to alternate responding to examples and explain that the student who is responding, they are the "player;" when they are checking their partner's responses, they are the coach. The coach will write the player's response in the appropriate space on the player's response sheet and provide feedback regarding the player's response (e.g. positive verbal reinforcement for accurate responses and corrective feedback for inaccurate responses.) For inaccurate responses, the coach provides feedback and the player attempts the question a second time. The first response is crossed out and the second response is recorded. The player first identifies each fractional part, then uses their cards to identify whether the first fractional part is greater than, less than, or equal to the second fractional part. After this, the player describes to the coach why the first fractional part is greater than, less than, or equal to the second one.

5.) Provide time for student questions.

6.) Signal students to begin.

7.) Monitor students as they work in pairs. Provide positive reinforcement for both "trying hard," responding appropriately, and for students using appropriate tutoring behaviors. Also provide corrective feedback and modeling as needed.

8.) Review student response sheets and note special difficulties individual students may be having and/or progress they are making.

2. Expressive Level

Purpose: to provide students with multiple opportunities for making meaningful connections between what they already know about concrete representations of fractions and comparing them to determine which fraction is greater than, less than, or if two fractions are equivalent.

Learning Objectives 1-4: Comparing fractions with like or unlike denominators using concrete materials (Area or Sets Model).

Planned Discovery Activities

Materials:

Students -

- planned discovery learning sheets - each sheet has three sections that prompts students to : 1.) represent different fractions that have selected denominators (e.g. "2," "4'" "6'" & "8") with concrete materials. At least two different fractions for each chosen denominator should be included (e.g. "1/2" & "2/2"; "2/4" & "3/4;" "3/6 & 5/6;" "5/8" & "3/8"); 2.) represent two fractions and respond whether one is greater than the other or if they are equivalent; 3.) develop “new” comparisons, at least two examples for “greater than,” two examples for “less than,” and two examples for “equal to.”

- appropriate concrete materials (circle pieces/fraction strips for Area Model; discrete counting objects like unifix cubes or beans for Sets Model).
- “word strips” that represent each fraction and the phrases “greater than,” “less than,” or “equal to” for student who have writing problems (if appropriate).
- pencils for writing.

Description:

Activity:

For comparing like denominators:

Students can work in groups, in pairs, or independently. Each student responds to a planned discovery-learning sheet. The sheet prompts them to do three primary things. First, they are prompted to represent different fractions that have selected denominators (e.g. “2,” “4,” “6,”, and “8”) with concrete materials. At least two different fractions for each chosen denominator should be included. (e.g. “1/2” & “2/2”; “2/4” & “3/4;” “3/6 & 5/6;” “5/8” & “3/8”). As students finish representing their fractions, they raise their hands so the teacher can check their responses (*or another student can check if students are working in pairs or small groups.) After a student’s responses are checked, then they move to the second section of the learning sheet. The second section prompts them to compare those fractions that have like denominators. For each “like denominator,” students compare them and then write whether one fraction is greater than the other or whether the two fractions are equivalent (e.g. “two-halves is greater than one-half; “three fourths is greater than “three-fourths.”) (*For students with writing problems, pre-made word strips can be used that have the appropriate fractions written as well as the phrases “greater than” and “equal to. Another accommodation would be to have students orally compare the fractions by raising their hand and reporting to the teacher or by telling their comparison to a peer.) After students have be “checked off” for each of the comparisons, they respond to the third section of the learning sheet. The third section prompts students to represent and show the comparison of at least two “new” comparisons showing “greater than,” two comparisons showing “less than,” and two comparisons showing “equal to.” Each “new” comparison must incorporate different fractions from those used in the previous section. The same “check off” procedure used for sections one and two can be used to verify student comparisons for section three.

For comparing unlike denominators:

The same process can be used as described for “like denominators” except that the second section of the planned discovery-learning sheet should include comparing unlike denominators. Likewise, the third section prompts students to develop “new” comparisons for unlike denominators.

Planned Discovery Activity Steps:

1. Develop Planned Discovery Activity Learning Sheet as described under *Materials*.
2. Distribute the Planned Discovery Activity Learning Sheet and provide clear directions for completing the activity, including appropriate behavioral rules.
3. Model how to complete one example for each of the three sections on the Planned Discovery Learning Sheet (and model appropriate behaviors as needed).
4. Provide students with appropriate materials (e.g. appropriate concrete materials).
5. Monitor students as they practice, providing appropriate corrective feedback, prompting student thinking, providing positive reinforcement, and modeling or cueing as needed.
6. At the conclusion of the activity, provide students with solutions to the Planned Discovery Activity Learning Sheet, especially Part 3. Elicit student examples for Part 3, providing students with appropriate feedback and model several examples provided by students. Emphasize *why* the examples represent "greater than," "less than," or "equal to" (e.g. the area represented by the numerator in relationship to the denominator.
7. Review student response sheets and note special difficulties individual students may be having and/or progress they are making.

Instructional Phase 3: Evaluation of Student Learning/Performance (Initial Acquisition through Mastery/Maintenance)

Continuously Monitor & Chart Student Performance

Purpose: to provide you with continuous data for evaluating student learning and whether your instruction is effective. It also provides students a way to visualize their learning/performance.

Materials:

Teacher –

- appropriate prompts if there will be oral prompts
- appropriate visual cues when prompting orally

Student –

- appropriate response sheet/curriculum slice/probe
- graph/chart

Description:

Steps for Conducting Continuous Monitoring and Charting of Student Performance:

1.) Choose whether students should be evaluated at the receptive/recognition level or the expressive level.

2.) Choose an appropriate criteria to indicate mastery.

3.) Provide appropriate number of prompts with a curriculum slice in an appropriate format (receptive/recognition or expressive) so students can respond. *Based on the skill, your students' learning characteristics, and your preference, the curriculum slice or probe could be written in nature (e.g. a sheet with appropriate prompts; Index cards with appropriate prompts), or oral in nature with visual cues (e.g. display concrete representations of fractions and prompt students to write or circle "greater than," "less than," or "equal to") or a combination of written curriculum slices/probes and oral prompts with visual cues.

4.) Distribute to students the curriculum slice/probe/response sheet/concrete materials.

5.) Give directions.

6.) Conduct evaluation.

7.) Count corrects and incorrects/mistakes (you and/or students can do this depending on the type of curriculum slice/probe used – see step #3).

8.) You and/or students plot their scores on a suitable graph/chart. A goal line that represents the proficiency (for concrete level skills, this should be %100 – 5 out of 5 corrects) should be visible on each students' graph/chart).

9.) Discuss with children their progress as it relates to the goal line and their previous performance. Prompt them to self-evaluate.

10.) Evaluate whether student(s) is ready to move to the next level of understanding or has mastered the skill at the abstract level using the following guide:

Concrete Level: demonstrates %100 accuracy (given 3 to 5 response tasks) over three consecutive days.

11.) Determine whether you need to alter or modify your instruction based on student performance.

Instructional Phase 4: Maintenance – Periodic Practice to Maintain Student Mastery of Skills

Purpose: to provide students the opportunity to review/practice math skills students has previously mastered.

1. Problem of the Day

Materials:

Teacher –

- a visual platform for displaying concrete objects that all students can clearly see (e.g. circle pieces with magnetic strips glued on the back and placed on chalkboard or dry-erase board).
- appropriate concrete materials to represent fractions.

Students -

- a sheet of paper to write responses on.
- a pencil for writing.

Description:

As students enter class or as the math period begins, display two concrete representations of fractions (e.g. circle pieces that represent "two-fourths" and "three sixths." Students must identify each fractional part and decide their relationship (e.g. greater than, less than, equal to). On a sheet of paper, students write their responses and also briefly explain why they chose their response (e.g. because the area of the circle covered by "two-fourths" and "three-sixths" is the same.). The teacher can quickly review the problem with students by asking for responses and providing appropriate feedback. Students can also hand in their response sheets and the teacher can review individual student responses. *Students can be invited to come up and run their fingers along the area of the fractional part or to take a closer look if helpful.