Instructional Plan

Abstract Level

Name of Math Skill/Concept: Division Process and Division with Remainders

Prerequisite Skills Needed:

1.) Concrete understanding of the division process.

2.) Ability to solve division equations using concrete materials.

3.) Ability to draw solutions for division problems.

4.) Ability to use the FASTDRAW Strategy to find the important information in division story problems, to set up a division equation, and to a draw solution.

5.) Ability to skip count.

6.) Mastery of multiplication facts/ability to use a times table key or calculator to figure multiplication facts.

Learning Objectives:

1.) Solve division story problems/equatons without the use of concrete objects or drawings.

Important Ideas for Implementing This Teaching Plan:

1.) This teaching plan provides description of instructional methods and activities that teach students to use the FASTDRAW Strategy and the DRAW Strategy to solve both division story problems and division equations. The plan also provides suggestions for how to help students increase their fluency for basic division facts.

2.) The three methods described to teach students how to "answer" division equations at the abstract level can all be helpful strategies for students. Whether you teach one or more than one method is dependent on the needs of your students. The order in which they are presented represents a natural progression from drawing solutions for division equations. The first strategy described, "visualizing pictures," can be an effective method for students with strong visual memory skills but may prove troublesome for those students who have trouble with visual memory or visual imagery. The second method, "repeated addition," can be very effective, particularly for students who understand the repeated addition process. Linking division to the multiplication process of repeated addition provides students a solid foundation to practice division at the abstract level. The third method is also an important link to the multiplication process but requires a more sophisticated understanding of what the multiplicands and the product represent. By explicitly teaching what they represent and associating this to the division process can be very empowering to students. However, students who have learning problems or who have limited prior knowledge of multiplication need very explicit instruction regarding these relationships.

3.) Students who have learning problems will need ample opportunities to practice solving division problems at the abstract level before they become fluent. For students who understand the concept of division but have difficulty remembering the steps in dividing, having continued access to the steps of the DRAW Strategy will allow them to independently solve division equations without continual teacher prompting regarding which step to do next. With multiple opportunities to respond, students' memory of the necessary steps will improve.

4.) Allow students to draw solutions as a "back-up" when needed. Providing students this alternative will allow them to work independently. Again, multiple independent practice opportunities are the best chance for students to move to abstract level success!

5.) Fading student use of both the DRAW Strategy and drawings is an important final step, however the fading process should occur only for those division facts they demonstrate ability to solve "abstractly." It is perfectly reasonable and sound instruction to encourage students to use cues for those division problems/facts they have not yet acquired at the abstract level, but to require students not to use cues for those division problems/facts they demonstrate acquisition or mastery of.

6.) Providing daily one-minute timings and charting student progress is an excellent method to increase students' fluency and mastery of division facts (See Instructional Phase 4 – Continuously Monitor and Chart Student Performance in this abstract level instructional plan).

Instructional Phase 1: Initial Acquisition of Skill/Concept – Teacher Directed Instruction

I. Teach Skill/Concept within Authentic Context

Description:

Meaningful contexts will continue to be used to teach the division process in the form of story situations that have relevance for 9-11 year old students. It is important always to provide story situations that have relevance to your students given their age, cultural backgrounds, and interests.

II. Build Meaningful Student Connections

Purpose: to help students build meaningful connections between what they know about drawing solutions to division problems and finding solutions without the use of drawings.

Materials:

Teacher –

- a visual display of the FASTDRAW Strategy.

- a visual display of the written learning objective: : "Use FASTDRAW to solve division story problems and division equations without drawing pictures." (*Highlight the phrase "without drawing pictures" to make it stand out.)

Description:

1.) **L** ink to students' prior knowledge of using FASTDRAW to solve division story problems and to draw solutions to division equations.

For Example:

You have learned to solve division story problems using a strategy. Who remembers the name of the strategy that helps us to solve division story problems? (Elicit the response, "FASTDRAW.") Yes, FASTDRAW is a strategy that helps us to solve division story problems. (Hold up a display of the FASTDRAW strategy.) What does the "FAST" in FASTDRAW help us to do? (Point to "FAST" and elicit the response, "find the important information in the story problem and set up a division equation.) That' correct, "FAST" helps us find the important information in the story problem and it also helps us to set up a division equation. What does the "DRAW" in "FASTDRAW" help us to do? (Point to "DRAW" and elicit the response, "it helps us solve the division equation by drawing pictures.") Excellent! The "DRAW" in the "FASTDRAW" Strategy helps us to solve division equations by drawing pictures.

2.) **I** dentify the skill students will learn

For Example:

For the next few days, we're going to learn how to use FASTDRAW to solve division story problems and division equations without drawing pictures. (Display and point to the written objective: "Use FASTDRAW to solve division story problems and division equations without drawing pictures.") What are we going learn to do? (Point to the written objective and elicit the response, "use FASTDRAW to solve division story problems and division equations without drawing pictures.") Good.

3.) **P** rovide rationale/meaning for

For Example:

Learning to solve division problems without drawing pictures will come in handy when you have to divide in a hurry. For example, at recess, you and your friends may want to play a game where you have to divide up into teams that have the same number of players on each team. Because you only have a limited amount of time for recess, you want to determine how many players on a team quickly so you can have as much time to play as possible. Being able to divide without drawing pictures will help you start playing faster.

III. Provide Explicit Teacher Modeling

Purpose: to provide students a clear teacher model of how to divide (with and without remainders).

Materials:

Teacher –

- a visual display of the FASTDRAW Strategy.
- appropriate story problems written so that all students can see it. *Color-coding can be faded at this point in instruction, but tell students that number phrases will no longer be color-coded.
- a visual platform to write where all students can see your writing.
- colored markers/pens/chalk

Students -

- FASTDRAW Strategy Cue sheet.

Description:

A. Break down the skill of solving division story problems/equations without the use of concrete objects or drawings.

*The same steps described in the representational level teaching plan for implementing the FASTDRAW Strategy can be used for implementing the strategy at the abstract level. The only difference is that you will teach students to "answer" the equation ("A" step in DRAW) without drawing pictures.

1.) Introduce story problem.

2.) Read the story problem aloud and then have students read it with you.

3.) Find the important information in the story problem and setting up an equation using the steps "FAST" from the "FASTDRAW" Strategy.

  3a. **F**ind what you are solving for.

  3b. **A**sk yourself, what is the important information (circle it).

  3c. **S**et up the equation.

  3d. **T**ie down the sign.

4.) Teach solving a division equation using the steps "DRAW" from the "FASTDRAW" strategy.

  4a. **D**etermine the sign.

  4b. **R**ead the problem.

  4c. **A**nswer (without drawing).

    - visualize drawings
    - use repeated addition

- use knowledge of multiplication facts

4d. **W**rite the answer.

5.) Model how to solve the story problem by relating the “answer” to the division equation back to the story problem context.

6.) Model how to solve division equations by repeating the steps in#4 and #5 at least two or three more times with different division equations.

B. Explicitly describe and model solving division story problems using FASTDRAW without drawing solutions.

*Use the same process as described in the representational level teaching plan. However, teach students to “answer” the division equation (during the “A” step of DRAW) without drawing pictures. Three examples of how you might cue students how to solve the division equation without drawing pictures is provided below. After reviewing each approach, decide which approach will meet the needs of your students best. You may want to teach all three approaches. If you decide to teach all three approaches, it is suggested that you teach them in the order presented because the skills needed by the student to successfully use each approach increase with each approach. These strategies should be modeled during the “A” step of DRAW. Previous to this step, you will have modeled using FAST to find what you are solving for, the important information, and to set up the division equation. You will have also modeled discovering the sign, and reading the problem. These instructional approaches also can be used with division equations that are not a part of a story problem context. Teach solving such equations using the DRAW strategy and implement the approaches described during the “A” step.

1.) Model how to solve division equations by visualizing drawings instead of actually drawing pictures.

1.) Introduce that you will be modeling how to “answer” division problems without drawing pictures.

For Example:

Now that we have read the problem, it is time to complete the ”A” step. What does the “A” in DRAW stand for? (Elicit the response, “answer, or draw and check.”) That’s right. We now need to answer the problem. You have learned to solve division equations by drawing pictures and that is one thing the “A” step suggests you do. However, I am going to show you how to solve the division equation without drawing pictures. You can always draw pictures if you are unable to solve it without them, but we are going to start learning to solve division problems without drawing pictures.

2.) Model answering the division equation without drawing, cueing students by “thinking aloud” the cognitive process for doing this.

- Model visualizing the dividend.

For Example:

Ok, I know I need to separate the dividend into groups based on the divisor. (Point to the dividend and the divisor as you say this.) Hmm, I my dividend is "ten." I need to separate or divide the dividend ten into groups of three. I know I need to separate "ten" into groups of three because my divisor is "three." (Point to the divisor.) If I drew the dividend "ten" then I would draw ten tallies or ten dots. Hmm, I think I can picture that in my head. I'll do that now. (Demonstrate thinking to yourself by putting your hand on your chin and looking up.) Ok, I can see ten tallies on a piece of paper. Everybody draw ten tallies on your paper. (Provide students enough time to draw their tallies.) What you see on your paper is what I see "in my head." Now, I want you to look at your tallies and then close your eyes. Try to picture those tallies in your head. When you have them pictured in your head, raise your hand. (Provide students time to do this.) Ok, now you see in your head what I see. I'll draw the ten tallies I saw in my head on the board. (Draw ten tallies.)

- Model visualizing the divisor by grouping the tallies into groups based on the divisor.

For Example:

The next thing I would do if I were drawing pictures to solve this problem would be to group the tallies into groups of three by circling the groups. Hmm, I could try doing visualizing this in my head, but I think I might have trouble. However, I know another way I could group the tallies in my head. I can visualize tallies in rows of three each until I have a total of ten tallies. I'll do that now. As I am visualizing the groups of tallies, I want you to draw them on your paper. Remember, you need to group you tallies into rows of three until you have ten total tallies. Your last row may have fewer than three tallies because there may be a remainder. (Demonstrate thinking to yourself by putting your hand on your chin and looking up while students draw their groups.) Ok, I see one group of three tallies, two groups of three tallies, three groups of three tallies and a fourth group that has only one tally. Therefore my answer is "three with a remainder of one." What did you draw? (Elicit several student responses and then draw what you "visualized.") This is what I visualized in my head (Point to the grouped tallies you just drew.) What is my solution? (Elicit the response, "three with a remainder of one.") That's right. The answer is three with a remainder of one.

- Review the visualization process to re-emphasize the cognitive cues used to solve the division problem.

For Example:

Now that I have solved the division problem without drawings, let's review what I did. When completing the "A" step in DRAW, I can answer the equation by visualizing the pictures I would draw if I were actually drawing pictures to find the answer. First, I visualize drawing tallies to represent the dividend. What did I visualize first? (Elicit the response, “the dividend.”) How many tallies did I visualize in my head? (Elicit the response, “ten.”) Yes, I visualized ten tallies because my dividend was ten. (Point to the dividend and then point to the ten tallies you drew to represent what you “saw in your head.”) Let’s practice picturing the ten tallies in our heads. Everybody look at their paper and then close your eyes. Try seeing what is on your paper in your head. (Provide students time to do this.) Raise your hand when you see the ten tallies in your head. Great job practicing how to make pictures in your head that represent pictures we draw on paper. Now, what is the next thing I did to solve this equation. (Elicit the response, “put the tallies in groups of three in your head.”) Yes, I knew I needed to group the tallies into groups of three because the divisor is “three.” Let’s practice picturing the groups of three tallies in our heads. Everybody look at the groups you drew on your paper and then close your eyes. Try seeing what is on your paper in your head. (Provide students time to do this.) Raise your hand when you see the groups of three tallies in your head. (Provide students time to do this.) How many groups of three tallies do you see? (Elicit the response, “three.”) Yes, there are three groups. Any tallies left over? (Elicit the response, “yes, one tally.”) Great job visualizing division in your heads! So, what is our answer? (Elicit the response, “three with a remainder of one.”) Excellent!

3.) Repeat this process at least three more times with a variety of division equations.

2.) Model how to use repeated addition to solve the division equation.

1.) Link to student prior knowledge of and experiences with repeated addition.

<u>For Example:</u>

Let me show you a problem you already know how to solve. (Display a repeated addition problem such as “5 + 5 + 5 = __.”) What is the answer to this problem/number sentence? (Elicit the appropriate response.) That’s right, the solution/sum is “fifteen.” How did you know this? (Elicit the response, “by adding five plus five plus five.”) Good. Let’s do this together by skip counting. (Point to each five as you and your students skip count. *Relate to grouped drawings of five tallies under each ‘5’ if you think additional cueing is needed for skip counting.) Great!

<u>For Example:</u>

5 + 5 + 5 = 15
“five” “ten” “fifteen”

2.) Model how to use skip counting to solve division equations by writing the divisor beside the problem in repeated addition fashion and then skip counting.

For Example:

Skip counting, when we add the same number several times, can help us do division without drawing pictures. Let me show you how. (Display a division equation with the dividend and divisor color-coded to help students distinguish them.)

$5\overline{)15}$ or 15 ÷ 5

Now, in order to solve this division equation, I can use skip counting. What is my dividend or total? (Elicit the response, "15.") Good. What is my divisor? (Elicit the response, "5.") Great! I need to find out how many groups of five can be made from fifteen. (Point to the divisor and then the dividend as you say this.) Hmm, how could I use skip counting to do this? Well, I know I have groups of five because "five" is my divisor. I could write a five over here to represent one group of five. (Write the number five to the right side of the equation or below the equation.)

5 15 5 15 ÷ 5

5

Ok, that gives me one group of five. "Five" is less than "fifteen," so I know I need at least one more group of "five." I'll write a second five. (Write another five next to the one you just wrote.) Ok, I have two groups of five.

For Example:

5 15 5 5 15 ÷ 5

5 5

Let me skip count to see how many I have. (Point to each "5" as you skip count.) Well, now I have a total of "ten" with my two groups of five. "Ten" is still less than "fifteen" so I can add another group of "five". (Write another "5.") I'll skip count again. (Point to each five as you skip count.)

5 15 5 5 5 15 ÷ 5

“five” “ten” “fifteen”

5 5 5

“five” “ten” “fifteen”

Now I have fifteen total. Three groups of five equal fifteen. Since I have three groups of five, (Point to each “5” and count, “one, two, three. Circling each five might be a helpful cue to some students since it replicates the process they used for drawing except now numbers represent the tallies/dots they once drew.) and that equals fifteen, then my quotient or answer to this division equation is “three.”

*For students with visual and/or spatial orientation problems, teaching them to draw several circles first or boxes first and then writing the number inside the circle can be a helpful cue. This technique can also help students with estimation because they can predict the number of circles or boxes they need to draw first and then check how close they were when they complete the skip counting process.

I can complete the last step of the DRAW strategy, “W” by writing my quotient or answer in the appropriate space. (Write “3” in the appropriate place. Color-code “3” to distinguish it as the “quotient.”)

$$\begin{array}{r} 3 \\ 5\overline{)15} \end{array}$$

$15 \div 5 = 3$

3.) Review the dividend, divisor and quotient by reviewing what each means and relate them to the repeated addition process.

For Example:

Now that we have found our answer. let’s review our problem and what each number means. Let’s read the problem first. (Read the problem and its answer aloud with the students, pointing to each number and symbol as you read it.) Great job! Now let’s review what each number and symbol means. (Point to the dividend.) What does this number represent? (Elicit the response, “the total.”) Good. The number fifteen represents the total. What do we call the total in a division problem? (Elicit the response, “dividend.”) Yes, we call the total the dividend. (Point to the word card displayed in the room reading “dividend.”). Ok, what does this symbol mean? (Elicit the response, “divide.“ Point to the “dividend” word card displayed in the room as needed for cueing or for re-emphasis of the written word.) That’s right, it means “divide.” What does the word divide mean? (Elicit the response, “to separate into equal groups.”) Great job! Divide means we separate the total into equal groups. Now, what does this number represent? (Elicit the response, the number of things in each group.) That’s right, it represents the number of things we have in each group. What is the name for the number of things in each group when we are solving division problems? (Elicit the response, "divisor." Point to the "divisor" word card displayed in the room as needed for cueing or for re-emphasis of the written word.) Right. We call the number representing how many in each group, the divisor. Alright, I'm going to read these parts of the problem again, and I want you to tell me what it all means. (Point to each part of the problem and read, "fifteen divided by five.") What does "fifteen divided by five" mean? (Elicit the response, "it means we are separating 'fifteen' into equal groups of five.") Excellent thinking! Now, the last number we haven't identified is the number "three." (Point to the number "3.") What does the number "three" represent. (Elicit the response, "the number of groups of five we can separate fifteen into.") That's right, "three" represents the number of groups of five we can separate "fifteen" into. What is the "division" name for the three, or the number of groups? (Elicit the response, "quotient." Point to the "divisor" word card displayed in the room as needed for cueing or for re-emphasis of the written word.) Yes, we call the number representing the groups the dividend can be separated into the "quotient." The number "three" is the "quotient."

4.) Repeat this process with at least three more examples. *The same process can be used for division problems that have remainders. When students get to the point where adding the divisor one more time results in a sum greater than the divisor, then model to students how to count on from the previous sum until they reach the value of the dividend. That number becomes the remainder.

3.) Model how to use knowledge of multiplication facts to solve division equations.

1.) Link to student knowledge of and experiences with multiplication facts.

<u>For Example:</u>

You know how to use skip counting to solve division problems. Skip counting is one way to multiply. You have already learned a lot about multiplication and multiplication facts/times tables. Like skip counting, we can use what we know about multiplication facts to solve division equations. Let's review some multiplication facts. (Display several multiplication facts on the chalk-board, dry-erase board, or overhead projector.) Alright, what does "4 times 2" equal? (Point to the multiplication fact and elicit the response, "eight." Continue this process for two more facts.) Great!

2.) Model the relationship of the product and the multiplicands in a multiplication fact.

<u>For Example:</u>

We know the answer to each of these multiplication facts, but what do these problems really mean? Well, let's take a look at one problem and see. I'm going to read this problem. (Read "4 x 2.") Now, in any math problem, the numbers and symbols mean something. This is true for multiplication problems as well. I can read this problem by saying "four times two equals eight." Another way to say it is "four groups of two is eight." (Point to each number and symbol as you say this, then rewrite the problem this way underneath the original fact.) I'd like for you to read this problem both ways with me. (Read the problem both "ways" with your students, pointing to each written representation of the problem.) Ok, let's look closer at this problem. (What does the number "four" mean? (Point to the number "4" and elicit the response, "it means there are four groups." Point to "four groups" if cueing is needed.) Yes, the number "four" means "four groups." Ok, what does the "two" mean? (Point to the two and elicit the response, "there are two in each group.") Good. The two represents two things in each of the four groups. So, what does the "eight" represent? (Point to the "8" and elicit the response, "the total.") Yes. Yes, eight is the total. Therefore, when we have four groups and two are in each group, we have a total of eight.

3.) Model the relationship of the product and multiplicands to the dividend, divisor, and quotient in a division problem.

- Display both a division problem and its associated multiplication fact and say what each problem means.

<u>For Example:</u>

$$\begin{array}{r} 5 \\ 4\,\overline{)\,20} \end{array} \qquad\qquad \begin{array}{r} 5 \\ \times\ \underline{4} \\ 20 \end{array}$$

Here we have two equations. (Point to the division equation.) What kind of equation is this one? (Elicit the response, "division.") That's correct. This is a division problem. I know it is a division problem because this sign represents/means division. (Point to the division symbol.) What kind of equation is this? (Elicit the response, "multiplication.") Yes, this is a multiplication problem. I know this is a multiplication problem because this sign represents/means "multiplication." (Point to the multiplication symbol) I'm going to write some words next to the numbers in each equation to make them more meaningful. (Write words/phrases next to each number in the problems that provide a meaningful context.)

$$\begin{array}{rl} & \text{5 groups} \\ \text{4 children} & \overline{)\,\text{20 total children}} \end{array} \qquad\qquad \begin{array}{rl} 5 & \text{groups} \\ \times\ \underline{4} & \text{children} \end{array}$$

20 total children

Although these are two different equations, division and multiplication, what we know about multiplication can help us solve division equations. Let's read each equation to find out what they mean. Let's read the division equation first. (Point to the division equation and read it aloud with your students, "twenty divided by four equals five.") Now, I'm going to read the same problem but use slightly different words. A total of twenty when separated into groups of four becomes five groups. (Point to the division equation and each number/symbol as you say the phrase and the words that correspond to each number/symbol.) I'll write this on the board. (Write the phrase on the board or display an already written phrase below the division equation. Color-code the words that correspond to each number/symbol. Point out to students how the colors in the words and the numbers show their relationship.).

<u>For Example:</u>

$$\begin{array}{rl} & \text{5 groups} \\ \text{4 children} & \overline{\big)\,\text{20 total children}} \end{array}$$

"A total of twenty children when separated into groups of four children each has five groups."

Ok, what does this division problem mean? (Point to the phrase on the board and elicit the appropriate response.) Great! The "twenty" is the total, the division symbol means to "divide" or to "separate the total," the "four" represents the number of things in each group when you divide or separate the total, and the "five" represents the number of groups of four that the total can be divided or separated into. (Point to each number and the words in the phrase that corresponds to it.) Let's review what each number and symbol means. (Review what each number and symbol means: e.g. What does the twenty represent? What does the division symbol represent?...)

We know what the division equation means and what each number and symbol means, now let's do the same for the multiplication problem. Let's read the multiplication equation. (Point to the multiplication equation and read it aloud with your students, "five times four equals twenty.") Good. Now, I'm going to read the problem again but I'm going to use slightly different words, just like I did with the division equation. Five groups of four equals a total of twenty. (Point to the multiplication equation and each number/symbol as you say the phrase and the words that correspond to each number/symbol.) I'll write this on the board. (Write the phrase on the board or display an already written phrase below the multiplication equation. Color-code the words that correspond to each number/symbol. Point out to students how the colors in the words and the numbers show their relationship.).

For Example:

5 groups
x 5 children
25 total children

"Five groups of four equals a total of twenty."

Ok, what does this multiplication problem mean? (Point to the phrase on the board and elicit the appropriate response.) Great! The "five" is the number of groups, the multiplication symbol means to "of"," the "four" represents the number of things in each group, and the "twenty" represents the total number of things in the groups altogether. (Point to each number and the words in the phrase that corresponds to it.) Let's review what each number and symbol means. (Review what each number and symbol means: e.g. What does the twenty represent? What does the division symbol represent?...)

4.) Explicitly model the relationship of the dividend in the division equation and the product in the multiplication fact.

For Example:

(Point to the dividend "20.") What does this number represent? (Elicit the response, "the total.") Yes, twenty is the total. What is the name for the total in a division problem? (Elicit the response, "dividend.") Good. (Point to the product, "20.") What does this number represent in this multiplication problem? (Elicit the response, "the total.") Yes, "twenty" in this problem represents the total as well. So, what does the "twenty" in this division problem (Point to the "20" in the division equation.) have in common with the "twenty" in the multiplication problem? (Point to the "20 in the multiplication problem) (Elicit the response, "they both are the total.") That's right, each is the total in their respective problems. So, the dividend in a division problem and the product/answer in a multiplication problem mean the same thing. They represent the total.

5.) Explicitly model the relationship of the divisor to the multiplicand that represents "how many in each group - "4."

For Example:

Now that we know the “twenty” in both the division equation and the “twenty” in the multiplication problem mean the same thing, let’s look at the other numbers in the two problems and see how they relate to each other. Let’s examine the number “4” in each problem. (Point to the “4” in the division problem.) What does this number represent? (Elicit the response, “the number of things in each group.”) Yes, the number “four” in this division problem represents the number of things that will be in each group as we divide/separate the divisor, “twenty,” into equal groups. Now let’s look at the number “4” in the multiplication problem. What does this number represent? (Elicit the response, “the number of things in each group.) That’s right, the “four” in this multiplication problem represents the number of things in each group.

6.) Explicitly model the relationship of the quotient to the multiplicand that represents number of groups - “5.”

For Example:

We know the “four” in both the division problem and the multiplication problem represents the number of things in each group. Now let’s examine the number “5” in each problem. (Point to the “5” in the division problem.) What does this number represent? (Elicit the response, “the number of groups.”) Yes, the number “five” in this division problem represents the number of equal groups that the dividend “twenty” can be separated into. What do we call the number that represents the groups the dividend can be separated into? (Elicit the response, “the quotient.”) Yes, the number “five” is our quotient and it represents the number of equal groups the dividend can be separated into. How many are in each of the five groups? (Elicit the response, “four.” Point to the divisor for additional cueing.) Good. Now let’s look at the number “5” in the multiplication problem. What does this number represent? (Elicit the response, “the number of groups.”) That’s right, the “five” in this multiplication problem represents the number of groups. The number five in the multiplication problem represents the same thing that the five in the division problem represent. (Point to “5” in both problems.) What do they both represent? (Elicit the response, “the number of groups.”) That’s right, the “quotient” in a division problem (Point to the “5” in the division problem>) and the first number in a multiplication problem (Point to the “5” in the multiplication problem.) both represent the number of groups. How many are in each group in the multiplication problem? (Elicit the response, “four.” Point to the “4” in the multiplication problem for additional cueing.)

7.) Model the relationship of the divisor and quotient in a division problem to the two multiplicands in multiplication problem.

For Example:

We now know what each number in the division problem and the multiplication problem represent, and we also know that each number in a division problem means has a “brother” or “sister” in a multiplication problem. I call them “brothers” and “sisters” because they are related. Let’s review these relationships.

(Review the similarities of the numbers in each problem- e.g. dividend and product; divisor and second multiplicand; quotient and first multiplicand.) Now, I want us to look at each problem again and compare them. I will re-write them and then I want you to look at them and tell me what you notice. (Re-write the same two equations.)

5 groups
4 children ⟌20 total children

5 groups
x 4 children
25 total children

What do you notice? (Elicit appropriate responses – e.g. "they have the same numbers;" the fives have the same color, the fours have the same color, the twenties have the same colors.) Excellent observations! I am going to write a symbol between the "four" and the "five" in the division problem that will actually turn it into a multiplication problem. When I do this, you will see how multiplication problems and division problems are related. (Write the "x" symbol between the "4" and the "5.")

x 5 groups
4 children ⟌20 total children

5 groups
x 4 children
20 total children

Now, examine the two problems again and tell me what you see that is similar. (Provided students time to examine the problems.) What do you see that is similar? (Elicit appropriate responses, "if you multiply five times four in the division problem, the answer is twenty;" "the dividend is the answer to five times four.") Great observations guys! When I look at this problem, I see that if I multiply the quotient, "five" (Point to the quotient.) by the divisor, "four," then the answer or product is "twenty." I know that this is true because when I look at the multiplication problem, I can see that "five times four equals twenty." (Point to the multiplication problem as you say this.) I can read my newly made "multiplication problem". (Point to the quotient, divisor, and dividend and read it as a multiplication sentence: "Five groups of four equals twenty.") I can also read my "original" multiplication problem and see that the same thing is true for it. (Point to each number of the multiplication problem and read it: "Five groups of four equals twenty.") Now that we know how to make a "multiplication" problem out of a division problem, I can really see why the numbers in these two different types of problems mean similar things. (Review each number and what it means for each type problem, re-stating how the corresponding number in the other type problem means the same thing.)

8.) Model using knowledge of multiplication facts to solve division equations.

For Example:

Now that we know how division problems and multiplication problems are similar, I am going to show you how to use your knowledge of multiplication to solve division problems. (Display a division problem that represents a multiplication fact and the corresponding multiplication fact.)

$$\begin{array}{r|l} & \text{? groups} \\ \cline{2-2} \text{5 children} & \text{25 total children} \end{array} \qquad \begin{array}{r} \text{5 groups} \\ \times\ \underline{5}\ \text{children} \\ \text{25 total children} \end{array}$$

9.) Model comparing the two equations and finding what is similar.

For Example:

Here I have a division problem and a multiplication fact that can help me solve the division problem. (Point to both problems. Ok, I know one is a division problem and one is a multiplication problem. (Point to each problem and say them aloud – “”twenty-five divided by five equals;” “five times five equals twenty-five.”) Even though one is a division problem and one is a multiplication problem, I now know from what we have learned that there are things in common between the two types of problems. Hmm, what can I find that is common between these two problems? Well, the two totals are the same. (Point to the dividend and the product.) Each total is “twenty-five.” Now, both division and multiplication problems involve groups and objects in those groups. I wonder how this knowledge can help me. Well, I can see that the number in each problem that represents the number of things in each group is the same. (Point to the “5” in each problem that represents the divisor and second multiplicand.) I know that in the division problem, “twenty-five” is to be divided into equal groups that have five things in each group. I know this because my dividend or total is “twenty-five and my divisor is “five.” (Point to the division problem and then point to the dividend and divisor as you say this.) The divisor, “five,” tells me how many things should be in each group. In the multiplication problem, I know this number five also means the number of things in each group. (Point to the “5” that represents the number in each group.)

10.) Model using the fact “5 x 5” to solve the division equation.

For Example:

Ok, how can I use what is the same in this division and multiplication problem to solve the division problem? Well, we already know how to write in a multiplication sign between the divisor and the quotient of a division problem. (Show students the previous problem where you modeled this.) By writing multiplying the divisor and the quotient, we found out that the answer/product was the dividend. Let me write in a multiplication sign here. (Write a multiplication sign in the appropriate place.

x ? groups

$5\text{ children}\overline{)\,25\text{ total children}}$

5 groups

x $\underline{5}$ children

25 total children

In this division problem, I don't know the quotient. (Point to where the quotient will be written.) That is what I need to solve for. However, I can still use what I know about multiplication to help me solve this problem. I already know from the multiplication fact that I have written over here that "five times five equals twenty-five." (Point to the corresponding multiplication problem, emphasizing the first multiplicand) Because I know this, I now know that if I write five for my quotient, then I will have the solution to my division problem. (Write "5" for the quotient.)

x 5 groups

$5\text{ children}\overline{)\,25\text{ total children}}$

5 groups

x $\underline{5}$ children

25 total children

How did I know the quotient should be five? (Elicit the response, "because five times five equals twenty-five; because the multiplication problem told you so.") That's right. I used the multiplication fact to help me solve the division equation.

11.) Review why the multiplication fact can help solve the division problem

<u>For Example:</u>

Now that we have solved this division equation by using its corresponding multiplication fact, let's review why we can use multiplication to solve division problems. We can do this by remembering what the numbers in each type of problem really mean. (*Review with students what each number in each type problem represents and how the corresponding numbers in each type problem represent the same thing. Use the written language to emphasize these relationships)

12.) Repeat this process with at least three more examples. Use multiplication facts that all students have mastered.

IV. Scaffold Instruction

Purpose: to provide students the opportunity to build their initial understanding of how to divide without concrete materials or drawings, and to provide you the opportunity to evaluate your students' level of understanding after you have initially modeled this skill.

Materials:

*Dependent on the skill you are Scaffolding Instruction for (See the materials listed for the specific skill you want to scaffold under Explicit Teacher Modeling).

Description:

*Scaffolding at the abstract level of instruction should occur using the same process as scaffolding instruction at the concrete and representational/drawing levels of instruction (See the description of Scaffolding Instruction in the Concrete Level Instructional Plan for this math concept.). The steps listed for each skill during Explicit Teacher Modeling should be used as structure for scaffolding your instruction.

A. Scaffold instruction using a high level of teacher direction/support. (Dependent on the needs of your students, you may want to continue to associate drawings to the abstract level rounding process during this phase of scaffolding. Move to the next phase of scaffolding only when students demonstrate understanding and ability to respond accurately to your prompts.)

B. Scaffold instruction using a medium level of teacher direction/support. (If you associated drawings with the abstract process for rounding while scaffolding using a high level of teacher direction/support, then do not include drawings during this phase of scaffolding. Move to the next phase of scaffolding only when students demonstrate understanding and ability to respond accurately to your prompts.)

C. Scaffold instruction using a low level of teacher direction/support. (Students should actually divide as you prompt them during this phase of Scaffolding Instruction.. Move students to independent practice of the skill only after they demonstrate the ability to perform the skill with limited prompting from you.)

Instructional Phase 2: Facilitate Acquisition to Mastery – Student Practice

*The student practice strategies described below can be used for both skills taught during initial acquisition through Teacher Directed Instruction. A detailed description for providing practice for one of the skills is provided below: Explicitly relate the place value of digits in one, two, and three digit numbers to where concrete materials are grouped on the place value mat.

1. Receptive/Recognition Level

Purpose: to provide students multiple opportunities to choose accurate solutions to division problems when give several choices.

Learning Objective 1: Solve division story problems/equations without the use of concrete objects or drawings – Solving division equations.

A. Self-Correcting Materials – Clothespin Division (adapted from Mercer & Mercer, 1998)

Materials:

Teacher –

- develop a variety of Clothespin Division Cards – Each card is separated into eight regions on the front and back. On the front of the card, a division equation is written in each region. On the back of the card, one of eight different symbols is written in each region (e.g. ♣ ♦ ♥ ♠ • * #). Cards can reflect certain division fact families, especially problematic division problems, or random division problems/facts dependent on the needs of your students. Each card is numbered, "1, 2, 3, …")
- a set of eight clothespins for each Clothespin Division Card. Each clothespin has the correct answer of one division problem on one side and the corresponding symbol of the division equation it answers on the other side.
- zip lock bags to store each card and set of clothespins. *Each bag can be numbered by card number to store corresponding clothespins while cards are kept altogether in a separate bag or other container.
- Answer Key that lists the problems and answers for each Clothespin Division Card.

Students -

- Clothespin Division Cards and corresponding clothespins.
- a response sheet (sheet of paper)
- a scratch sheet of paper for working out problems if needed
- pencil

Description:

Activity:

Students work with square or rectangular shaped cards divided into eight regions. On the front side of the card, division problems are written in each region. On the back of each card are eight different symbols with one symbol written in each region. Each card is numbered at in the center or in one of the corners in an alternate color. Students right the number of each card they respond to on their response sheets. Each Clothespin Division Card has a set of eight clothespins. Every clothespin has the correct answer to one division equation written on one side and the corresponding symbol from the back of the card written on the other side of the clothespin. Students choose the clothespins that "answer" each division equation and clip them to the card in the appropriate region. When students have answered all of the equations, they flip the card over. If the symbol on a clothespin matches the symbol on the back of each card, then the student knows they solved the equation correctly. As students finish a card and check their answers, they record the number they got correct on their response sheet next to the card number. The teacher circulates the room and monitors students as they work, providing positive reinforcement, providing specific corrective feedback, and answering questions as appropriate. Students can exchange cards as they finish. The teacher evaluates student performance by reviewing their response sheets.

Idea for Monitoring Student Performance

You and your students can keep track of how they do with each numbered card by writing the number correct on an individual chart that lists the different card numbers (e.g. graph paper can be used where each column represents a different card and the card's number is written at the top or bottom of a column). Each row represents days the student practices with the Division Clothesline Cards. Dates can be written to indicate the day of practice. Each time a student responds to a card, he/she (or you) can record the total number correct on their chart for each card they respond to. When the student gets eight correct three days in a row, they can put a star next the number that represents that card. Students can "visualize" their progress and this process can be an efficient way for you to quickly monitor student progress, particularly if each "card number" represents particular division facts/problems. You can quickly see which fact families students are becoming proficient with at the receptive/recognition level.

Self-correcting Materials Steps:

1.) Introduce self-correction material.

2.) Distribute materials.

3.) Provide directions for self-correcting material, what you will do, what students will do, and reinforce any behavioral expectations for the activity.

4.) Provide time for students to ask questions.

5.) Model responding/performing skill within context of the self-correcting material.

6.) Model how students can keep check their responses.

7.) Have students practice one time so they can apply what you have modeled. Provide specific feedback/answer any additional questions as needed.

8.) Monitor students as they work.

9.) Provide ample amounts of positive reinforcement as students play.

10.) Provide specific corrective feedback/ re-model skill as needed.

11.) Encourage students to review their individual response sheets.

13.) Review individual student performance record sheets.

II. Expressive Level

Purpose: to provide students multiple practice opportunities to solve division problems in a motivating format.

Learning Objective 1: Solve division story problems/equations without the use of concrete objects or drawings – Solving division equations.

Instructional Game – Division Basketball

Materials:

Teacher –

- sets of cards that represent division problems with increasing levels of difficulty. Lay up cards have the easiest problems, 10 foot jump shot cards have more difficult problems, and 3 point shot cards are the most difficult. The front side of each card has the type of shot written on it, the problem the offensive player answers, and the problem(s) the defensive player answers to "block." The answer to each problem is written on the back of each card. Two "block" problems are written for "lay up" cards. One "block" problem is written for "10 foot jump shots." No "block" problems are written on "3 point shot" cards. (*One set of problems for each type of shot can be made on sheets of paper so that problems and answers for each card can be cut out to fit the size of a 4x5 note-card. The master sheets can then be copied to make as many sets as needed. Problems and answers are glued to the front and back of a each note-card and a letter is written (a-z). Note-cards can be laminated to protect them.

For Example:

Front of Lay-Up Card

Lay-Up **A**

Shot	Block	
8	9	12
÷ 2	÷ 3	÷ 6

Back of Lay-up Card

Shot	Block	
8	9	12
÷ 2	÷ 3	÷ 6
4	3	2

- response sheets that have three columns labeled "Lay-Up," "10 foot jump shot," and "3 point shot."
- drawing of basketball court on chalkboard/dry-erase board or poster board posted in front of room.

Students -

- three decks of Division Basketball cards (Lay-Up, 10 foot jump shot, 3 point shot).
- response sheet
- scratch piece of paper for solving problems
- pencil

Description:

Activity:

Students work in pairs or small groups. Each pair or small group has a drawing of a basketball court on tag board (a larger basketball court could be drawn on the chalkboard/dry-erase board or on a poster-board and placed in the front of the room as an alternative to providing a smaller version for each student pair). Additionally, each student pair or small group has three sets of cards. Each set represents division problems that increase in difficulty (See description of cards under "Materials.") The easiest set of cards are for "lay-ups." The next most difficult set of cards are "10 foot" jump shots. The most difficult set of cards are "3 point shots." Students take turns pulling on card from one of the three sets of cards. The student who "has the ball" chooses whether to shoot a "lay up" for one point, a "10 foot jump shot" for two points, or a "3 point shot" for three points. The player "shoots" by answering the division problem on the card. The player receives the appropriate number of points if they answer the problem correctly. The player or team on defense has the opportunity to "block" a lay-up or 10 foot jump shot by responding to the "block" division problem(s) also on the front of side of those cards. If the "defense" answers the "block" division problem(s) correctly, then the offensive player does not receive points for their shot. If a player chooses to answer a "3 point shot" card, the defense cannot block that shot. After both the player on offense and the player on defense answer their problems, they check their answers by turning the card over. Each player records the letter of the card they answered under the column labeled "lay-up," "10 foot jump shot," or "3 point shot" along with the points they made (both offensive shots and block attempts). Teacher monitors students providing positive reinforcement, specific corrective feedback and answering questions as appropriate. Teacher reviews individual student response sheets to evaluate student performance.

Instructional Game Steps:

1.) Introduce game.

2.) Distribute materials.

3.) Provide directions for game, what you will do, what students will do, and reinforce any behavioral expectations for the game.

4.) Provide time for students to ask questions.

5.) Model how to respond to the card prompts.

6.) Provide time for students to ask questions about how to respond.

7.) Model how students can keep track of their responses.

8.) Play one practice round so students can apply what you have modeled. Provide specific feedback/answer any additional questions as needed.

9.) Monitor students as they practice by circulating the room, providing ample amounts of positive reinforcement as students play, providing specific corrective feedback/ re-modeling skill as needed.

11.) Play game.

12.) Encourage students to review their individual response sheets.

13.) Review individual student response sheets to determine level of understanding/proficiency and to determine whether additional modeling from you.

Instructional Phase 3: Evaluation of Student Learning/Performance (Initial Acquisition through Mastery/Maintenance)

1. Continuously Monitor & Chart Student Performance

Purpose: to provide you with continuous data for evaluating student learning and whether your instruction is effective. It also provides students a visual way to "see" their learning.

Materials:

Teacher –

- appropriate prompts if they will be oral prompts
- appropriate visual cues when prompting orally

Student –

- appropriate response sheet/curriculum slice/probe
- graph/chart

Description:

Steps for Conducting Continuous Monitoring and Charting of Student Performance:

1.) Choose whether students should be evaluated at the receptive/recognition level or the expressive level.

2.) Choose an appropriate criteria to indicate mastery.

3.) Provide appropriate number of prompts in an appropriate format (receptive/recognition or expressive) so students can respond.

- At the abstract level of understanding, the most efficient format for a curriculum slice/probe is written (e.g. student responds in writing to written prompts).

4.) Distribute to students the curriculum slice/probe/response sheet/concrete materials.

5.) Give directions.

6.) Conduct evaluation.

7.) Count corrects and incorrects/mistakes (you and/or students can do this depending on the type of curriculum slice/probe used – see step #3).

8.) You and/or students plot their scores on a suitable graph/chart. A goal line should be visible on each students' graph/chart that represents the proficiency (near %100 accuracy with two or fewer incorrects/mistakes) and a rate (# of corrects per minute) that will allow them to be successful when using that skill to solve real-life problems and when using the skill for higher level mathematics that require use of that skill.

9.) Discuss with children their progress as it relates to the goal line and their previous performance. Prompt them to self-evaluate.

10.) Evaluate whether student(s) is ready to move to the next level of understanding or has mastered the skill at the abstract level using the following guide:

> *Abstract Level:* demonstrates near %100 accuracy (two or fewer incorrects/mistakes) and a rate (# of corrects per minute) that will allow them to be successful when using that skill to solve real-life problems and when using the skill for higher level mathematics that require use of that skill.

11.) Determine whether you need to alter or modify your instruction based on student performance.

2. Additional Assessment Activity Appropriate For This Math Skill/Concept

Purpose: to assess where student understanding of the rounding process is "breaking down."

Flexible Math Interview/C-R-A Assessment

Materials:

Teacher –

- appropriate concrete materials for dividing (See Concrete Level Instructional Plan – Explicit Teacher Modeling.).
- appropriate examples for assessment (division problems)
- paper to record notes.

Description:

Have students solve division problems using concrete materials, by drawing, and without concrete materials or drawings. Ask students to explain their answers as they respond. Note where in the division process students "break down;" both at what level they begin having difficulty and at what point within that level of understanding they demonstrate misunderstanding/non-understanding. Based on where students demonstrate difficulty, provide explicit teacher modeling at that level of understanding and for the particular sub-skill they are having difficulty with. As the student demonstrates understanding, scaffold your instruction until they are ready to practice the skill independently. As students demonstrate mastery of the skill at that level of understanding, then provide explicit teacher modeling at the next level of understanding. Follow this process until students demonstrate mastery at the abstract level.

Key Ideas

1.) Students who demonstrate difficulty at the abstract level of understanding may have “gaps” in their understanding that can be traced back to their representational/drawing level of understanding or even their concrete level of understanding. By providing additional teacher modeling at the level their “gap” in understanding began and then moving them from a concrete-to-representational-to-abstract level of understanding, you can assist students to become successful at the abstract level of understanding.

2.) Sometimes students demonstrate difficulty at the abstract level because they did not receive enough practice opportunities at the concrete and representational/drawing levels. The drawing level is a very important step for these students. Some students need continued practice drawing solutions and associating their drawings to the abstract symbols and the mental processes necessary to perform at the abstract level.

3.) Some students understand the concept, but have difficulty remembering the steps involved to perform the skill at the abstract level. Providing students with cues they can refer to as they practice at the representational/drawing and abstract levels of instruction is very helpful (e.g. DRAW Strategy). Such cueing provides them the independence to practice. Multiple practice opportunities translate into repetition, and repetition enhances memory. The use of instructional games and self-correcting materials are an excellent way to provide students with multiple opportunities to solve division problems.

4.) Helping your students build their fluency for solving division facts can also increase their abstract level problem-solving efficiency. Providing daily one-minute timings and charting student performance is an effective way to do this. It is important to communicate with students what their “learning pictures” (charts) mean and to set short-term achievable goals. Seeing “what” they are striving for and seeing their progress as they move toward a goal is very motivating for children! (See the description of the instructional strategy “Continuous Monitoring and Charting Student Performance” for more information. This description can be found by clicking on “Instructional Strategies” on the main menu bar found on your left panel.

5.) Enhancing the “meaningfulness” of abstract equations can also aid students who are having difficulty achieving mastery at the abstract level both by providing them a deeper level of conceptual understanding and by enhancing their memory of the problem-solving process. One approach you might try is to reinforce what the numbers and symbols mean using language. By modeling language (and encouraging students to use their own language) that describes what each number and symbol represents, students can gain a deeper level of understanding of the “abstract process” they are struggling to master.

For Example:

12 ÷ 4 = ___

12 “CD’s” shared among 4 friends becomes ? CD’s per friend

Provide your students multiple opportunities to use their language as they practice solving equations. As students practice, they have the opportunity to associate “meaning” to the abstract process.

Instructional Phase 4: Maintenance – Periodic Practice to Maintain Student Mastery of Skills

*Maintenance activities at the abstract level of understanding should include concrete and representational/drawing experiences as well as “abstract” (numbers and symbols only) experiences. By “re-visiting” previous concrete and representational/drawing experiences, students reinforce the conceptual understanding they acquired during those phases of instruction. Including “language experiences” during these maintenance activities, where students describe their solutions, also reinforces conceptual understanding students established during their concrete and representational/drawing experiences.

Purpose: to provide students with opportunities to maintain their level of mastery of solving division story problems and division equations by drawing.

1. Instructional Games & Self-Correcting Materials

Materials:
*Dependent on the Instructional Games or Self-Correcting Materials you implement.

Description:
*Periodically provide students opportunities to practice division with and without remainders via self-correcting materials and instructional games. This can be done via “centers,” in small groups, or as a whole class. Include opportunities to solve division problems with concrete materials and by drawing in addition to abstract level practice opportunities. Even though students master a concept/skill at an abstract level, providing maintenance practice opportunities using concrete materials and by drawing reinforce their conceptual understanding. (*See the descriptions for “Instructional Games” and “Self-Correcting Materials” for more information of how to implement these student practice strategies.)

2. Problem of the Day

Materials:
Teacher –

- a written prompt on the chalkboard, dry-erase board, or overhead projector (e.g. a division problem or division story problem) or a concrete/drawing example representing a solution to a division equation (e.g. solution to a division problem that includes a remainder).

Students -

- paper and pencil to record their responses

Description:

Teacher presents a “problem of the day” that focuses on a particular skill or conceptual understanding of solving division story problems and/or division equations. The problem can be written in nature where students solve the problem with concrete materials, by drawing, or at the abstract level only. Students can also be challenged to develop a story problem for an already solved division equation. The “problem of the day” is displayed as students enter the room or as the period begins. Students are asked to “solve” the problem and provided necessary directions. After an appropriate amount of time, the teacher and the students “talk through” the problem and its solution. Students can individually describe how they approached the problem. Specific positive verbal reinforcement is provided by the teacher as well as specific feedback regarding misunderstandings students may have. Teacher notes students who seem to be having difficulty for the purpose of reviewing/re-modeling appropriate skills and concepts.

Ideas for Prompts:

1.) Display the concrete or drawing representation of an equation as well as its solution and ask students to represent the equation and the solution using only numbers and symbols.

2.) Display an equation and ask students to represent the equation and the solution with concrete materials or drawings.

3.) Display a concrete, drawing, or abstract representations of an equation and have students develop a story problem for that equation.

4.) Display an equation and solution with concrete materials, by drawing, or with only numbers and symbols with one part of equation missing (e.g. one of the mixed numbers being added) and ask students to determine the missing part.