$\mathcal{N a m e}$ of $\mathcal{M a t h} S \mathrm{Kill} /$ Concept: $\mathcal{T}$ wo digits by one digit multiplication with regrouping using drawings.

Prerequisite SKills Needed:

- Multiplication fact families
- Understanding that multiplication is repeated addition
- Grouping of ones and tens
- Multiplying tens
- Multiplying two digit by one digit numbers without regrouping
- Ulse of place value mats and materials
- Identification of factors and product in a multiplication problem,
- Multiplying two digit by one digit numbers with regrouping using concrete objects


## Learning Objectives:

1. Solve two digits by one digit multiplication problems with regrouping using drawings to show the process of repeated addition.
2. Solve two digits by one digit multiplication problems with regrouping by using drawings to show fownumbers are regrouped.
3. Solve two digits by one digit multiplication problems with regrouping by using drawings to illustrate the algoritfimic process.

Important Ideas for Implementing This Teaching Plan:

1. In this teaching plan there are three objectives. The first objective (1) provides a transition from using concrete objects to using representations of these objects and emphasizes the process of repeated addition that was used in the concrete lesson. The second objective (2) introduces the concept of the algorithmic process while emphasizing the regrouping step. The third objective (3) continues using the $\mathcal{D R A}$ ( strategy with the algorithmic process and provides a transitional stage for moving students to the abstract level. It is important that students progress through all four phases of the teaching plan for Objective 1 PRIOR to starting activities outlined under Objective 2. Again, students should progress through all four phases of the teaching plan to fully master Objective 2 before moving to Objective 3.
2. Ulse story problems to provide context
3. Color cue marks for ones and tens
4. This lesson is based on the premise that students have had prior experience with using the DRAM strategy and $\mathcal{F I} \mathcal{N} \mathcal{D} S$ trategy to solve mathematic al problems. If they have not, then some additional teaching on the reason and use of strategies will be needed

5. This teaching plan is written for the skill of two digits by one digit multiplication with regrouping. Following the same sequence of steps as outlined below, students would have had previous experience with using base ten materials and place value mats to complete multiplication problems involving multiplying tens by 1 digit and 2 digit by 1 digit numbers without regrouping.
6. This teaching plan outlines the steps for teaching two digits by one digit multiplication with regrouping. If additional instruction at the concrete levelis needed for problems involving multiplying three digit numbers by one digit or two digit factors, the same sequence of steps as outlined in this plan can be followed.

Instructional Phase 1: Initial Acquisition of Skill/Concept-Teacher Directed Instruction

Teach Skill/Concept within Authentic Context

Description: Links are made to the concrete experiences and the contexts used at the concrete level.
(See Build Meaningful Student Connections). Problem solving is emphasized (See Explicit Teacher Modeling)

Build Meaningful Student Connections

Purpose: to assist students to build meaningfulconnections between what they knowabout problems using concrete objects and using drawings to solve two digit by one digit multiplication problems with regrouping.

* The following description is an example of howyou might implement this instructional strategy for Le arning Objective 1. A similar process can be used for the other learning objectives in this plan.

Learning Objective 1: Solve two digits by one digit multiplication problems with regrouping using drawings to show the process of repeated addition.

Materials:
Teacker -

- Packages of items
- Area for visual dis play,
- Place value mat,
- Base ten blocks and ten sticks,
- Poster of $\mathcal{D R A}$ strategy

Description:
1.) Link to students'prior knowle dge of multiplying two digit by one digit numbers with regrouping using concrete objects and using the $\mathcal{D R A}$ W strategy to solve problems.

## For Example:

We have been working on math problems that involve regrouping with multiplication. We have been using counting 6 locks, ten sticks and place value mats to help us solve these problems. We have been figuring out problems like how many foot 6 all cards we would have if we had 3 packs of cards with 15 cards in each pack (show items).
2.) I dentify the skill students will le arn: Multiply two digits by one digit numbers with regrouping using drawings. For Example:

Today will we le arn how to use the $\mathcal{D R A W}$ strategy to solve these problems. We have used the $\mathcal{D R A}$. strategy to help us work problems before. Who can tell me what the $\mathcal{D}$ stands for? (Point to poster). What about the R $\mathcal{R}$ The $\mathcal{A}$ and the $\mathcal{W}$ ? Right, that is the $\mathcal{D R A} \mathcal{A}$ strategy. Today instead of using base ten 6 locks, ten sticks and place value mats (show items), we are going use the DRAW strategy and use lines and circles to solve the se multiplication problems.I ust like we have been doing, we are going to solve multiplication problems that involve regrouping.
3.) $\underline{\underline{P}}$ rovide rationale/me aning for solving two digit by one digit multiplication problems with regrouping.

## For Example:

It's not always convenient for us to carry around cubes, sticks, and our place value mats. By learning to use the $\mathcal{D R A W}$ strategy, we can solve multiplication proble ms with regrouping more quickly.

Provide Explicit Teacker Modeling

Purpose: to provide students with a clear teacher model of how to solve two digits by one digit multiplication problems with regrouping using drawings.

Learning Objective 1: Solve two digits by one digit multiplication problems with regrouping using drawings to show the process of repeated addition.

Materials:
Teacher -

- Place value mat, 6locks, ten sticks,
- Visual display board,
- Three different colors of markers,
- $\mathcal{T}$ wo sentence strips: "number of groups", "number of items in each group"
- Posters with mnemonics of $\mathcal{D R A}$ ( $\operatorname{strategy\text {,}}$
- Group of cookies in 6ags,

Description:
A. Break down the skill of solving two digits by one digit multiplication problems with regrouping using drawings to show the process of repeated addition.

1. Introduce a meaningfulcontext.
2. Introduce/review $\mathcal{D R A} \mathcal{W}$ strategy steps.
3. Ulse the $\mathcal{D R A}$ W strategy to Discover the sign.
4. Use the $\mathcal{D R A} \mathcal{W}$ strategy and Read the problem:
5. Ule the $\mathcal{D R A} \mathcal{W}$ strategy and Answer:

Draw a place value mat.
Draw number of groups.
Draw number of items/group.
Count one lines and trade groups of 10 .
6. Use the $\mathcal{D R A}$. strategy and Write the answer:

Count the ones lines and write the answer.
Count the ten lines and write the answer.
7. Repeat the multiplication problem.
B. Explicitly describe and modelfow to multiply two digits by one digit numbers with regrouping using the $\mathcal{D R} \mathcal{A} \mathcal{W}$ strategy to show the process of repeated addition.
1.) Introduce a meaningfulcontext.

Show concrete items represented by written problem

## For Example:

Today I want to buy 4 bags of these cookies. There are 17 cookies in each bag. (Show item). I want to know fow many cookies I will have all together. I am going to write that problem here on the board. I have 4 bags of 17 cookies and I want to see how many cookies I have all together. (Point to problem as you repeat the numbers).
2.) Introduce/revie w DRAW strategy steps.

Cue students to minemonic for the strategy

## For Example:

Remember that a strategy is like a tool. I ust like we use tools to help us build houses, we use strategies to help us comple te math problems. Today we are going to use the $\mathcal{D R} \mathcal{A} \mathcal{W}$ strategy to help us solve our multiplication problems. Each leter in the word $\mathcal{D R A} \mathcal{W}$ is a toolthat we can use to help us remember what to do when we solve multiplication problems. $\mathcal{D}$ stands for $\mathcal{D}$ iscover the sign. R is for Read the problem. A is for $\mathcal{A n s}$ wer 6y drawing and $\mathcal{W}$ is $\mathcal{W}^{2}$ rite the problem. (Display poster or other visull of memonic and point to eachletter as you review the steps).
3.) Discover the sign.

Circle the sign in the problem
For Example:
The first thing we need to do it to discover the sign. Here is my sign (circle it). It is a multiplication sign. A multiplication sign tells me that I am going to have a certain number of groups (point to the 4) and I have a certain number of the items in each group (point to17). I want to put all 4 groups together to get my answer (point to space for product). I want to see how many cookies I would have all together if I bought 4 bags of 17 cookies.
4.) Read the problem

- Cue students to mnemonic for the strategy
- Model how to identify which number is the factor that represents the number of groups and which is the factor that represents the number of elements in each group.
- Associate concrete items with factors
- Verbally and visually labeleach factor


## For Example:

Well, we've discovered the sign, the next thing we need to do is READ the problem. I know that it is multiplication, so when I read the problem, I am going to need to decide which number or factor shows the number of groups and which factor shows the number of items in each group. In this problem, I have 4 bags of cookies, so $I$ have 4 groups. 4 is the first factor in this problem. (Labelthe 4 with "number of groups".) The other factor is the number that tells the number of items in each group. In my example, each bag has 17 cookies, so 17 is the other factor.
(Labelthe 17 with the "the number of items in each group").
5.) Answer by drawing.

- Cue students to mnemonic for the strategy
- Link concrete items (place value mats, Gase ten 6locks, ten sticks) to drawings For Example:

I've discovered the sign and I've read the problem, now I want to $\mathcal{A N S}$ WER the problem. I am going to draw circles and lines to get the answer. We have been using place value mats, base ten blocks and ten sticks to figure out answers (show items). Now, we are going to draw the answers using circles and lines. Instead of using place value mats, we are going to draw a place value mat (demonstrate on board). And, instead of using string, 6 locks and ten sticks, we are going to use circles and lines. We are going to use circles instead of string and we are going to use short lines (demonstrate on Goard) for ones, and long lines(demonstrate on board) for tens. What are we going to use for ones? Right! And what are we going to use for tens? -Right again. Okay, let's do this problem. Let's find out how many cookies I will have all together if I buy 4 bags of 17 cookies each.

- $\quad$ Draw a place value mat.
- Cue students to links between drawings and place value objects
- Prompt student responses to features of place value mat
- Color code labels (ones, tens) of place value mat


## For Example

We fave been using place value mats to figure out the answers, now we are going to draw mats on our paper. Do you remember when we have drawn mats on our papers before? Good! The first thing we do is draw a $\mathcal{T}$. Then we will labelour $\mathcal{T}$. That $\mathcal{T}$ looks like our place value mat (I Clustrate with mat). It has two columns just like our mat does. Our place value mats'columns are labeled. What is this first column (point to ones column) Right! The Ones column. I am going put an $O$ at the top of this column on my $\mathcal{T}$ to show that this is the ones column. What is this column on our place value mat? Right again! It is for the Tens. So, what do you think I am going to put at the top of this cofumn on my $\mathcal{T}$ ? Right a $\mathcal{T}$ to stand for tens. Now I have drawn a place value mat. I have two columns, ones, and tens. (Use mat to illustrate similarity between drawing and place value mat).

- Drawnumber of groups:
- Drawcircle to shownumber of groups
- Link drawings to items in problem
- Cue students to link between drawings and previously used objects


## For Example:

I have 4 bags of cookies with 17 cookies each. 4 is my first factor (point to number in the written problem). What does it tellme? (Elicit student response). Right! It tells me fow many bags of cookies I have. I have 4 bags of cookies. When we used the place value mats, we used string to show our groups. (Iffustrate with mat). Since I am drawing, I am going to drawcircles to show fowmany groups I fiave. How many circles sfould I draw? (Elic it student response).

- Drawnumber of items/group
- Link drawings to items in problem
- Cue students to link between drawings and previously used objects
- Drawlines in each circle to correspond to the number of tens and the number of ones in the second factor.
- Color code one lines and ten lines
- Ulse think alouds and questions to modelfow to represent second factor using drawings For Example:
$\mathcal{N}$ Now that I ve shown that I have 4 groups, what do youthink I need to do next? Right, I need to show that I have 17 cookies in each Gag. I need to look at my other factor (point to 17 in written problem on Goard). When we were using our place value mats, we decided how many ones and how many tens we re in the other factor, and put blocks and ten sticks in each circle. Let's see if I remember how to do that. $\mathcal{H m m m}$, how many ones are in 17? Right, 7. So on my place value mat, I am going to put 7 6locks in the ones column in each circle. Now I need to look at the number of tens in 17. Howmany tens are in seventeen? Right! There is one ten in 17 , so I am going to put a ten stickin the tencolumn in each circle. (Iflustrate using concrete materials). Now I have shown that I have 17 items in each of the 4 groups. Well, if I want to $\mathcal{D R A}$ ' my answer, I wonder what I could do? You are right! I could use lines! I could use one lines (point to example) for 6 locks and ten lines (point to example) for ten sticks. I have 17 cookies in each 6ag, so I how many one lines do I need to draw in the ones column? (Elicit student responses). Right, I need to put 7 lines in the ones columnfor each circle. Howmany ten lines do I need to draw in the tens column? Right, I need to draw one ten line in the tens column for each circle. Now I have shown that I have 17 cookies in each bag by drawing.
- Count the number of ones represented in the circles. Trade groups of 10 from the ones to the tens column.
- Make groups of ten by crossing out the one lines.
- For each group of 10 , draw a ten line at the top of the ten column on the place value mat.
- Ulse questions to prompt students to estimate
- Color code one and ten lines
- Cue students to link between drawing and previously used objects


## For Example:

$\mathcal{N}$ Now I have represented the 17 cookies in each bag bydrawing one ten line and 7 one lines in ach circle. $\mathcal{B} u t$, we want to find out how many cookies we have all together. We need to add all of our groups of cookies together. Do youremember what we did when we used our place value mat and counting 6locks? Right! We counted all the blocks in the ones column and made as many groups of ten as we could.
(Iflustrate using concrete materials). I want to do that bydrawing, so I am going to count all the lines in the ones column. Each time I count a line, I am going to cross it out. (Ilfustrate with sample on board). By crossing it out, it let's me knowthat I have already counted that line. When I get to ten, I am going to put a ten line at the top of the tens column on my mat. Let's start counting. We have $1,2, \ldots 10$. We fave marked off one group of 10 , so I am going to make a ten mark here at the top of the ten column. Let's see, do you think we will get another group of 10 ? Well, let's see. 1,2..10. You were right! We did make another group of $10 . \mathcal{N}$ ow we have 2 ten lines at the top of our ten column. I wonder if I can make any more groups of 10 ? $1,2 \ldots 8$. Nope. -1 have 8 ones left.
6.) Write the answer

- Cue students to mnemonic for strategy
- Modelfiow to count the ones lines and write the answer.
- Modelfow to count the ten lines and write the answer.
- Use color coding to cue links between drawings and numbers
- Link to previously used concrete materials
- Prompt students to count any tens moved over from the ones column
- Ulse think alouds and questions to prompt students to identify the number shown by all ten lines


## For Example:

$\mathcal{N}$ (ow that I have drawn my solution, I need to count my lines and write my answer. I am going to count how many lines I have in the ones column and then I am going to count fowmany ten lines I have in the tens column. Remember when we used our place value mats, we first added up the blocks we had in the ones column (Iffustrate with concrete materials) and then we added up all the sticks we have in the tens column (illustrate with concrete materials). How many lines do $I$ have in the ones column? Right, We have $1,2 \ldots 8$ ones, so $I$ am going to write a 8 here in the ones column. (Ilfustrate by writing an 8 at the bottom of the ones column color coded). Let's see how many ten lines we have. Well, I had drawn one, two, three, four lines in each circle to show that we fad 17 cookies in each bag. I've added two more ten lines from the groups of $10 I$ made in the ones column. $S$ o, how many total ten lines do we have? Right! We have 1,2,3,4 plus 2 more, for a total of 6 all together. I have 6 tens all together, so $I$ am going to write a 6 here in the tens column.
7.) Repeat the multiplication problem, saying and writing the answer that is shown.

- Review problem
- Model reading answer from drawing


## For Example:

I have 6 tens and 8 ones. 6 Tens are 60 and plus 8 ones makes 68 . I know that the answer to this problem is 68 . So, if I have 4 Gags of 17 cookies, (point to problem $4 \times 17$ ) how many cookies will I have all together? RIGHTI! 68 So, I can write that $4 \times 17=68$.
8.) Repeat the steps at least two more times with different numbers.
9.) PRIOR to moving to Learning Objective 2, continue following the teaching plan for Objective 1 by next
 $\mathcal{M A I N} \mathcal{N} \mathcal{E N} \mathcal{A} \mathcal{N} C \mathcal{E}$.

Learning Objective 2: Solve two digits by one digit numbers with regrouping using drawings to show how numbers are regrouped.

* PRIOR to teaching this objective, the student should have successfully demonstrated independent mastery of the preceding objective

Materials:
Teacher.

- Visual display board,
- Three different colors of markers,
- $\mathcal{T}$ wo sentence strips: "number of groups", "number of items in each group"
- Posters with mnemonics of $\mathcal{D R A} \mathcal{W}$ and $\mathcal{F I} \mathcal{N} \mathcal{D}$ strategy,
- Groups of balloons in 6ags

Description:
A. Break down the skill of multiplying two digits by one digit numbers with regrouping using drawings to show how numbers are regrouped.
1.) Ulse the $\mathcal{D R A}$ W strategy to $\mathcal{D}$ iscover the sign.
2.) Use the $\mathcal{D R} \mathcal{A} \mathcal{W}$ strategy and Read the problem:

- Use the $\mathcal{F I} \mathcal{N} \mathcal{D}$ strategy to find factors'place values
3.) Ulse the $\mathcal{D R A W}$ strategy and Answer:
- Draw a place value mat.
- Draw answer for multiplying the ones
- Count one lines and trade groups of 10 .
- Draw answer for multiplying the tens
4.) Ule the $\mathcal{D R A} \mathcal{A}$ strategy and Write the answer:
- Count ones lines and write the answer
- Count ten lines and write the answer
5.) Repeat the multiplication problem.
B. Explicitly describe and model how to multiply two digits by one digit numbers with regrouping using the DRAW strategy to emphasize the regrouping step in the algorithmic process.
1.) Discover the sign.
- Show concrete items to bring relevance to problem
- Identify features of the problem
- Cue students to mnemonic for strategy
- Circle the sign in the written problem


## For Example:

Today I have a special problem. I want to plan a birthday party and I want to make sure that I have enough of everything I need. I particularly want to make sure that I have bought enough water balloons because I want to play a game with them. I will need to have lots, and lots of water balloons at the birthday party. I have 4 bags of Galloons and there are 14 balloons in each bag. I am going to write my problem fere, 14 x 4 . The first step in the DRAW strategy is to discover the sign. What is the sign in this problem? Right! It is a multiplication sign. It means that I have a certain number of groups (4) and I have certain number of items in each group (14) and I want to see how many items I have all together. I need to combine all 4 bags of 14 balloons to see how many I have all together.
2.) Read the problem.

- Cue students to mnemonic for strategy

O Model how to identify which number is the factor that represents the number of groups and which is the factor that represents the number of elements in each group.

- Associate concrete items with factors

For Example:
We've discovered that we have a multiplication sign. That means that we will need to figure out which factor tells me how many groups I have. Which number is it? Right! IT is 4 , because that tells me that $I$ have 4 groups or bags of Galloons. Which factor tells me how many balloons I have in each bag? Right again! 14 tells me how many I have in each group. In this problem, I have 14 balloons in each bag.

- Ulse the $\mathcal{F I N} \mathcal{N}$ D strategy to identify the place value of the factors.

O Reviewstrategy with students

- Model how to find the column by cueing students verbally and vis ually

O Modelfiow to insert the Ts between columns
O Color code labels ( $O$ and $\mathcal{T}$ ) when naming the columns in the written problem

## For Example:

We know that in this problem we want to find out how many birthday balloons we would have if we bought 4 packages of balloons and each package had 14 balloons in it. When you look at the problem that I have written on the board, remind me one more time which factor tells me how many groups I have? (Elicit student response). Right! The 4. And which ones tells me how many I have in each group? Right the 14. So I have written $14 \times 4$ for my problem. It will help me when I write my answer for this problem if I use the $\mathcal{F I} \mathcal{N} \mathcal{D}$ strategy (point to poster and review mnemonic). Remember, just like the $\mathcal{D R A W}$ strategy, the $\mathcal{F I} \mathcal{N} \mathcal{D}$ strategy gives me tools to help me when I solve a problem. Each letter stands for a different step or tool. The first step I am going to do is Find the columns in my problem. Let's see.I have the 4 ones and 1 ten in this factor (point to written problem) and I have 4 ones in this factor (point to written problem) so I have two columns-a ones column and a tens column (point to problem). The I stands for Insert the $\mathcal{T}$ 's. So I am going to Insert the $\mathcal{T}$ 's between my columns. Now I need to go to the third step -I need to $\mathfrak{N a m e ~ t h e ~ c o l u m n s . ~ I ~ w i l l ~ p u t ~ a n ~} O$ over the columnfor the one lines and a
$\mathcal{T}$ over the column for the ten lines. I have everything set up, so when I solve my problem, I ll Ge able to Determine the place value of my answer, and say it.
3.) Ulse the $\mathcal{D R A} \mathcal{A}$ strategy and $\mathcal{A n s w e r}$ :

- Draw a place value mat.
- Reviewmnemonic
- Color code labels (ones, tens) on place value mat


## For Example:

I discovered the sign and read the problem, now what am I going to do? Right! I am going to answer by drawing. What is the first thing I am going to draw? (Elicit student response). You all have learned this well. I need to draw a place value mat. I am going to drawa $\mathcal{T}$. Then I am going to label the columns.

- Draw answer for multiplying the ones.
- Demonstrate fowanswer was drawn in previous lesson (see Learning Objective 1).

O Model how to multiply the numeral in the ones column of the top factor by the bottom factor.

- Cue students to place value of factors
- Color drawings to match color codes of ones and tens labels


## For Example:

We fave beendrawing each group using one and ten lines, but now I am going to show you a shorter way to do it. Let's look at what we have beendoing. We have shown that we have 14 balloons in each Gag by drawing the number of ones and the number of tens in each group of 14 balloons. We have 1 Gag of Galloons here, so what should we draw in the ones column? - In the tens column? Now what should I do to show the 14 Galloons in the second bag? And the third? And the fourth? $\mathcal{N}$ (ow we have 4 groups of 14 . (Ilfustrate on board). We will need to combine these groups all together to get our answer. That is how we have been doing the problems.

Another, quicker way we cando this is to figure out how many ones we have in each group, and then how many tens we have in each group (point to written problem on board). We start with the ones. If we look at $14 \times 4$, what number is the top factor? Right 14 . What number is in the ones column in 14? Right the 4. How many ones are there in one group of 14 ? Correct, there are 4 . But we have 4 groups of 14 . I need to see how many ones we would have if we combined all the ones from all 4 groups together. I am going to multiply the 4 ones in $146 y$ our bottom factor, 4 . (Point to written problem). What is $4 \times 4$ ? You're on top of it! $4 \times 4=16$. If I combine all 4 groups of 14 , there are 16 ones in all the groups. I am going to put 16 marks in the ones column on my place value mat.

O Count ones lines and trade groups of 10 .

- Link trading groups of 10 to drawings in previous lesson

O Modelfow to make groups of ten from the one lines in the ones column.

O Model how to drawa ten line in the ten column of the place value mat for each group of 10 ,

- Continue to use color coding

O Prompt students to estimate

## For Example:

$\mathcal{N}$ ow I have drawn how many ones we have all together, we need to check and make sure that we don't have some tens in the ones column that need to be moved over. How have we beendoing that? Right! We have counted and crossed out each line. Each time we get to 10 we are going to make a ten line at the top of the ten column on the place value mat. I am also going to drawa line at the top of the tens column on our problem to help us when we count and write our answers. Let's see, I have 16 lines in the ones column. I wonder if I can make any groups of 10 ? You think I can? Show me how many groups of 10 youthink I can make? Well, let's see. 1,2..10. There is one group of ten. I am going to draw a line at the top of the tens column on my place value mat. Do you think I ll have any more groups of 10 to move over? $\mathfrak{N}$ o? Well, let me finish counting and see. 1,2...6. You were right; I can't make any more groups of 10 .

- Draw answer for multiplying the tens.

O Modelfow to multiply the numeralin the tens column of the top factor by the single digit, Gottom factor

O Cue students to place values of factors

- Color code ten lines


## For Example:

$\mathcal{N}$ (ow that I ve multiplied the bottom factor, 4 and the ones in my top factor, I need to move to my tens column and do the same thing. What number is the top factor? Right 14 . What is the number in the tens column? Right 1. How many tens do I have in 14 ? Right I have 1 ten. $\mathcal{B I U T}$ I have 4 groups of 14, so I am going to see how many tens I would have if I combined all the tens from all 4 groups of 14. I am going to multiply 1 ten by 4 (point to written problem). What is 1 x 4 ? Exactly! IT is 4 . I am going to put 4 ten lines in the tens column to show this.
4.) Use the $\mathcal{D R A} \mathcal{A}$ strategy and $\mathcal{W}$ rite the answer:

O Modelfow to count the number of ones lines and write the answer in the ones column.
O Modelfow to count the number of ten lines and write the number in the tens column.

- Continue to use color coding for ones and tens
- Link the drawing solution to the written problem


## For Example:

$\mathcal{N}$ ow I am ready to count and write my answer. I am going to count my ones first. How many lines do $I$ have in the ones column? Right 6.SoI am going to write a 6 here at the bottom of my ones column of my place value mat and I am going to write a 6 here under my ones column of my problem. I need to count my tens now. I had 4 lines when $I$ multiplied the 4 tens in each group by the 4 groups, 6 ut I have added one more ten. So, I have 1,2,..5. 5 tens. I am
going to write a 5 at the bottom of my tens column of my place value mat, and I am going to write a 6 fere under my tens column of my problem.
5.) Repeat the multiplication problem, saying the answer that is shown.

- Review problem
- Modelreading answer from drawing

For Example:
I wrote 56 here as my product. Uling the $\mathcal{F} I \mathcal{N D}$ strategy, let's determine how many ones I fave. I fiave 6 ones and how many tens? Right, I have 5 tens.I have 6 ones and 5 tens. 5 tens are 50 plus 6 ones makes $56 . I$ know that the answer to this problem $4 \times 14$ is 56 . If $I$ buy 4 bags of 14 balloons each, I will have 56 balloons in all. I think $I$ will have plenty of balloons for my party.
6.) Repeat at le ast 2 times using different number combinations.
7.) PRIOR to moving to Learning Objective 3, continue following the teaching plan for Learning Objective 2



Learning Objective 3: Solve two digits by one digit multiplication problems witf regrouping by using drawings to illustrate the algoritfimic process.

* PRIOR to teaching this objective, the student should have successfully demonstrated independent mastery of the preceding objective.

Materials:
Teacher.

- Visual display board,
- Three different colors of markers,
- Two sentence strips: "number of groups", "number of items ineachgroup"


Description:
A. Break down the skill of multiplying two digit by one digit numbers with regrouping using drawings to illustrate the algorithmic process.

1. Use the $\mathcal{D R A}$ whategy to Discover the sign.
2. Use the $\mathcal{D R A}$ ( strategy and Read the problem:

- Ulse the $\mathcal{F I} \mathcal{N} \mathcal{D}$ strategy to identify factors'place value

3. Use the DRAW strategy and Answer:

- Draw a place value mat.
- Draw answer for multiplying the ones
- Draw answer for multiplying tens

4. Use the $\mathcal{D R A} \mathcal{W}$ strategy and $\mathcal{W}$ rite the answer:
5. Count ones lines and write the answer.
6. Count ten lines and write the answer.
7. Repeat the multiplication problem,.
B. Explicitly describe and model how to multiply two digits by one digit numbers with regrouping using the DRAW strategy with an algoritfmic process.
1.) Discover the sign.

- Identify features of problem
- Cue students to mnemonic for strategy
- Prompt students to estimate
- Verbally and visually link numbers in story context to written problems

For Example:
We have been working hard on multiplication problems, and each time we are learning something that helps us do them a little easier. Today we are going to continue using the $\mathcal{D R A}$, strategy, $\mathcal{B U I}$ we aren't going to $\mathcal{D R A} \mathcal{A}$ as much as we have been. We are moving up to "BIG LEAGUES" and soon we will be doing these problems just by multiplying the numbers Let's use the $\mathcal{D R A}$ AW strategy and see how we canlearn to do that. I have a problem on the Goard (point to problem $13 \times 6$ written on board). I've decided to shop today to get sodas for the snack bar at the Gall field. I want to get about 75 sodas (Write number on board and circle it.) Each pack of soda has 6 cans in it (point to number in written problem). I 'm going to buy 13 packs (point to number written on board). The first step in the $\mathcal{D R A} \mathcal{A}$ strategy is to discover the sign. What is the sign in this problem? Right! It is a multiplication sign. It means that I have a certain number of groups (13) and I have certain number of items in each group (6) and I want to see how many items I have all together. I need to combine all 13 packs of 6 sodas to see how many I have all together. Do you think I'll have enough? Well, let's see.
2.) Read the problem.

- Cue students to mnemonic
- Modelfow to identify which number is the factor that represents the number of groups and which is the factor that represents the number of elements in each group.

We've discovered that we have a multiplication sign. That means that we will need to figure out which factor tells me fow many groups I fave. Which number is it? Right! It is 13, because that telfs me that I five 13 packs of soda. Whicf factor tells me how many sodas I have in each pack? Right again! 6 tells me how many I have in each group. In this problem, I fave 6 sodas in each pack.

O Ulse the $\mathcal{F I} \mathcal{N} \mathcal{D}$ strategy to identify the place value of the factors.

- Cue students to mnemonic
- Modeluse of strategy

O Ulse think alouds and questions to prompt students to strategy steps
For Example:
We know that in this problem we want to find out how many sodas I would have if I bought 13 packs of sodas and each packhas 6 sodas in it. I have written $13 \times 6$ for my problem. I am going to use the $\mathcal{F I} \mathcal{N D}$ strategy to helpme figure out what to multiply in this problem. Remember that eachletter stands for a different step or tool. What should I do first? Right, Find the columns in my problem. Let's see. I have the 3 ones and 1 ten in this factor (point to written problem) and $I$ fave 6 ones in this factor (point to written problem) so I have two columns - a ones column and a tens column (point to problem). Now I am going to Insert the t's betwe en my columns. Then what do I need to do? Rigft, I need to $\mathcal{N}$ ame the columns. I will put an $O$ over the column for the one lines and a $\mathcal{T}$ over the column for the ten lines. I have everything set up, so when I solve my problem, I th be able to Determine the place value of my answer, and say it.
3. Use the $\mathcal{D R A} \mathcal{W}$ strategy and $\mathcal{A n s}$ wer:

- Draw a place value mat.
- Reviewmnemonic
- Modelflow to drawmat


## For Example:

$I$ discovered the sign and read the problem, now what am I going to do? Right! I am going to answer by drawing. What is the first thing I amgoing to draw? (Elicit student response). You all are on top of things! I need to draw a place value mat. I am going to drawa $\mathcal{T}$. $\mathcal{T}$ fen $I$ am going to labelthe columns.

- Draw answer for multiplying the ones

O Modelfow to multiply the numeralin the ones column of the top factor by the single digit, 6ottom factor.

- Cue students to one and tens in product.

○ Modelfow to represent the answer by drawing ones and ten lines on the place value mat.

- Color code to show differences in place value

Look at the problem I have written on the board. 13 x 6 . I want to multiply 13 by 6 . Let's look at 13. Which number is in the ones column? Right the 3. We want to multiply the ones first, so we will multiply $3 \times 6$. (Write problem on board). That will tell us how many ones we have from all the groups together. What is $3 \times 6$ ? You are correct, it is 18 . We have been putting 18 marks in the ones column of our place value mat and then seeing howmany groups of ten we could make. But there is a quicker way to show 18 . Show me how many ones are in 18 ? Right! 8 (underline 8 in written problem). Show me fow many tens are in 18 ? Right! 1 (Underline 1 in written problem). Instead of making 18 marks in the ones column, and then trading over a group of 10 , I am going to show 18 by making 8 marks in the ones column, and putting one ten line in the tens column.

- Draw answer for multiplying tens.
- Modelfiow to multiply the numeralin the tens column of the top factor by the single digit, bottom factor.
- Cue students to place values of factors
- Model how to represent tens on place value mat
- Color code to show differences in place value


## For Example:

I have multiplied the 3 ones of the top factor by the bottom factor 6 (point to numerals in written problem) and I got 18 . To show that 18 , fias 8 ones and 1 ten, I put an 8 fere in the ones column, and a ten line here at the top of the ten (point to place value mat). Now I need to move to my tens column and do the same thing. Look at the problem I have written on the 6oard. 13 x. 6. Let's look at 13 . What is the number in the tens column? Right 1. Howmany tens do I have in 13 ? Right I have 1 ten. We will multiply $1 \times 6$ (point to written problem). That will tell us how many tens we have from all the groups together. What is $1 \chi 6$.? (Point to written problem) Right! It is 6. I am going to put 6 lines here in my tens column of my place value mat.
4.) Ulse the $\mathcal{D R A W}$ strategy and Write the answer:

- Modelfow to count the number of ones lines and write the answer in the ones column.
- Modelfow to count the number of ten lines and write the answer in the tens column.
- Continue to use color coding for ones and tens
- Link drawing solution to algorithmic process for written problem


## For Example:

$\mathcal{N}$ ow I am ready to count and write my answer. I am going to count my ones first. How many lines do I have in the ones column? Right \& So I am going to write an $\mathcal{B}$ here at the bottom of my ones column of my place value mat. Let's look at problem on the board. I multiplied the number in the ones column of the top factor (point to 3) by the 6ottom factor (point to 6). So I multiplied $6 \times 3$ and got 18 . Eighteen fias 8 ones (point to lines of place value mat) and one ten (point to ten line at the top of the ten column on the place value mat). I am going to write an 8 here under my ones column of my problem, and put a ten line at the top of my ten column of my problem. I need to count my tens now. I had 6 lines when I multiplied the ten in the top factor by the bottom factor. $\mathfrak{A N} \mathcal{N} \mathcal{D}$, I moved over one
ten when I multiplied $6 \times 3 . S$, I have $1,2, . .7$. 7 tens. I am going to write a 7 at the bottom of my tens column of
my place value mat.
I am going to write a 7 here under my tens column of my problem.
5.) Repeat the multiplication problem, saying the answer that is shown

- Review problem
- Model how to read the answer from drawing

For Example:
I wrote 78 here as my product. Uling the $\mathcal{F I} \mathcal{N} \mathcal{D}$ strategy, let's determine how many ones and tens I have. I have 8 ones and how many tens? Right, I have 7 tens. 7 tens are 70 plus 8 ones makes $78 . I$ know that the answer to this problem $6 \times 13$ is 78 . If I buy 13 packs of 6 sodas each, I will have 78 sodas in all. I will have just enough soda for the snack bar.
6.) Repeat at least 2 times using different number combinations.

Scaffold Instruction

Purpose: to provide students an opportunity to build the ir initial understanding of how to solve two digit by one digit multiplication problems with regrouping using drawings.

Materials:

* Dependent on the skill (See materials listed for the specific skill under Explicit $\mathcal{T}$ eacher Modeling).

Description:
Scaffolding at the representational/drawing levelof instruction should occur using the same process as scaffolding instruction at the concrete levelof instruction (See the description of Scaffolding Instruction for "using concrete objects to solve two digit by one digit multiplication problems with regrouping"in the Concrete Level Instructional P(an). The steps used during Explicit Teacher Modeling should be used as structure for scaffolding your instruction.
$\mathcal{H I} \mathcal{G} \mathcal{H}$
$\mathcal{M E D I}$ UIM
LOW

1. Scaffold instruction using a high levelof teacher direction/support ( ${ }^{*}$ Dependent on the needs of your students, you may want to continue to associate concrete materials with drawings at this levelas described under Explicit Teacher Modeling.) *Move to the next phase of scaffolding only when students demonstrate understanding and ability to respond accurately to your prompts.
2. Scaffold instruction using a medium levelof teacher direction/support (*If you associated concrete materials with drawings while scaffolding using a high levelof teacher direction/support, then do not include concrete materials during this phase of scaffolding). ${ }^{*}$ Move to the next phase of scaffolding only when students demonstrate understanding and ability to respond accurately to your prompts.
3. Scaffold instruction using a lowlevelof teacher direction/support (*Students should actually draw as you prompt during this phase of Scaffolding Instruction.). *Move students to inde pendent practice of the skill only after they demonstrate the ability to perform the skill with limited prompting from you.

Instructional Phase 2: Facilitate Acquisition to Mastery - Student Practice

Receptive/Recognition Level

Purpose: To provide students with multiple practice opportunities to multiply two digit by one digit numbers with regrouping using the drawings to illustrate the algorithmic process.

Learning Objective 3: Multiply two digits by one digit numbers with regrouping using the drawings to illustrate the algoritfmic process

Structured Peer $\mathcal{T}$ utoring
Materials:
Teacher -

- Sample of problem sheets
- Prompt sheet and response sheet to use when introducing and modeling activity. Students.
- Sets of problem sheets. -Eack sheet will be divided vertically. On one side will be a multiplication problem. On the other side will be three drawn solutions, one of which is the correct solution.
- Prompt sheets,
- Response sfieets,
- Pencils

Activity:
$S$ tudents will work in pairs. Each student will have a turn being a coach and being a player. The coach will have 10 problem sheets and a prompt sheet. The prompt sheet will illustrate the steps to solving the problem (see breakdown of steps in Explicit Teacher Modeling). The player will have a piece of paper with 10 boxes. Each box will be numbered 1-10 and will have 2 blanklines in it. The coach will show the player one of the problem sheets. The player will need to choose the solution to the problem. The coach will turn the sheet over, check the player's answer, and put a 5 on the first 6 lankline in the box if the player gets the answer right the first time. If the player does have the correct response, the coach will review the sequence of steps with the player. The player will have a second attempt to choose the correct solution, with the coach providing prompts. After the player correctly chooses the solution, the coach puts a 2 on the second blankline in the box. After the player has completed a set of 10 problems, the coach and player total the player's points and thens witch roles using a different set of problem sheets.

Structured Peer $\mathcal{T}$ utoring $S$ teps:
1.) Select pair groups and assigneach pair a place to practice (try to matcf students of varying acfievement levels if possible).
2.) Review directions for completing structured peer tutoring activity and relevant classroom rules. Practice specific peer tutoring procedures as needed (see step \#4).
3.) Modelfow to perform the skill(s) within the context of the activity before students begin the activity. Model both what the coach does (e.g.reads the questions/prompts on the learning sfeet; check answers using number card; provide corrective feedback; record points) and how the player responds (e.g. choosing the solution, using the mnemonic). Prior to starting this activity, the teacher will introduce the activity and model how:
a. $\quad \mathcal{A}$ player selects a problem sfieet, and reviews the strategy.
6. A coach watches the player and provides feedback and positive reinforcement.
c. A coachuses the prompt sheet to assist the player to rethink fis/her answer.
d. A player accepts feedback and reinforcement from the coach.
e. A player retfinks an answer that is not correct.
f. A coacfie-checks the answer and provides feedback and positive reinforcement.
g. One or both partners cansignal the teacher if a question arises.
6. Partners will switcfiroles once the player fias completed one set of 10 problems.
4.) Divide the practice period into two equal segments of time. One student in ach pair will be the player and the other student will be the coach.
5.) Provide time for student questions.
6.) Signal students to begin.
7.) Signal students when it is time to switch roles.
8.) Monitor students as they work in pairs. Provide positive reinforcement for both "trying fard," responding appropriately, and for students using appropriate tutoring befiviors. Also provide corrective feedback and modeling as needed.
a. Circulate around the room to ensure that all pairs are actively engaged.
6. Set individual goals for students and monitor progress towards those goals.
c. Provide corrective feedback and positive reinforcement to coaches and players.
d. Collect response sheets to trackstudent progress.
e. Provide whole-group review with one or more problems after all pairs have finished.

Expressive Level

Purpose: to provide students multiple opportunities to solve two digit by one digit multiplication problems with regrouping by using drawings to illustrate the algorithmic process.

Learning Objective 3: Solve two digits by one digit multiplication problems with regrouping by using drawings to ilfustrate the algorithmic process.

Self-Correcting Materials

Materials:
Teacher -

- Sets of note-cards or problem sheets with multiplication problem on front and answer drawn on back.
$\mathcal{N}$ (ote cards sfould be numbered (e.g. 1-10).
- Sets of learning sheets with boxes corresponding to the numbers on note cards. Each learning sheet will have a place for students to record their correct and incorrect responses . "C" (Correct) and " $\mathcal{H}$ " $(\mathcal{H} e(p)$. Students.
- A set of note cards
- $\quad \mathcal{A}$ set of learning sfieets
- Pencilfor drawing/writing

Description:
Activity:
Each student will have a set of note cards and learning sheets. Students pick a note card and solve the problem by drawing the answer in the corresponding box on their learning sheets. The students check their
answers by turning the card over where the correct drawing appears along with the answer to the written problem. Students mark their answer as correct or incorrect.

Self-Correcting Materials Steps:

1. Introduce self-correction material.
2. Distribute materials.
3. Provide directions for self-correcting material, what you will do, what students will do, and reinforce any Gefiavioral expectations for the activity.
4. Provide time for students to askquestions.
5. Model performing skill (e.g. choosing card, finding appropriate box on le arning she et, drawing ans wer).
6. Modelfowstudents canchecktheir responses.
7. Have students practice one time. Provide specific feedback/answer any additional questions as needed.
8. Monitor students as they work.
9. Provide ample amounts of positive reinforcement.
10. Provide specific corrective feedback. Remodelskill as needed,
11. Encourage students to review the ir individual le arning sheets, write the totalnumber of correct
responses next the "C" (Correct) and do the same for the " $\mathcal{H}$ " ( $\mathcal{H e}(p)$.

Instructional Pfase 3: Evaluation of Student Learning/Performance (Initial Acquisition througf
Mastery/Maintenance)

Continuous Monitoring $\mathcal{F}$ Charting of Student Performance

Purpose: to provide the teacher with continuous data for evaluating student learning and whether your instruction is effective. It also provides students a way to visualize the ir learning/progress.

Materials:
Teacher.

- Appropriate prompts if they will be oral prompts
- Appropriate visualcues when prompting orally

Students:

- Appropriate response sfieet/curriculum slice/probe
- Grapficfart

Description:
Steps for Conducting Continuous Monitoring and Charting of $S$ tudent Performance:

1. Choose whether students should be evaluated at the receptive/recognition level, the expres ive level, or both.
2. Choose appropriate crite ria to indicate mastery.
3. Provide appropriate prompts in an appropriate format (receptive/recognition or expressive) so students can respond:

- Based on the skill, your students'Learning characteristics, and your preferences, the curriculum slice or probe could be written in nature (e.g. a sheet with appropriate prompts; index cards with prompts), or oral in nature with visualcues (e.g. teacher shows severaldrawings/choices on overhead then prompts students to say which drawing shows the correct solution for the problem), or a combination of both (e.g. teacher shows problem and then prompts students to circle which of severaldrawings shows the correction solution).

4. Provide students with the materials to complete each task.
5. Provide directions on how to complete each task.
6. Conduct evaluation.
7. Count corrects and incorrects (you and/or students cando this dependent on type of curriculum slice/probe used).
8. You and/or the students will plot the ir responses on a suitable chart. A goal line that represents proficiency should be visigle on each student's chart. For representationallevelof understanding this should be $100 \%$. for $8-10$ trials.
9. Talk with children about their progress as it relates to.the goalline and the ir previous performance. Prompt them to self evaluate (Did you use the $\mathcal{D R A} \mathcal{W} / \mathcal{F} I \mathcal{N}(\mathcal{D}$ strategy?)
10. Evaluate whether student(s) is ready to move to the next levelof understanding or has mastered the skill by demonstrating $100 \%$ accuracy for $8-10$ trials over 2-3 days.
11. Determine whether you need to alter or modify your instruction based on student performance.
$\mathcal{A d d i t i o n a l}$ Assessment Activity Appropriate For $\mathcal{T}$ fis Math $\mathcal{S k i l l / C o n c e p t ~}$
$\mathcal{F l e} \chi$ ible Math Interview

Purpose: to provide the teacher with additional diagnostic information in order to checkstudent understanding, and plan and/or modify instruction accordingly.

Materials:
Teacher -

- Appropriate multiplication problems

Students.

- Paper, pencils

Description:

With individual students or in small groups, the teacher will have students draw solutions to given multiplication problems. The teacher will askstudents to explain how the ir drawings show the solution to the problem. The teacher should note errors or misconceptions while the student is "te aching", but the teacher should not stop the student for correction purposes. By having the student complete the entire explanation, the teacher will gain a better understanding of the student's thinking. The teacher confers with students regarding specific errors or misconceptions afterwards.

Instructional Phase 4: Maintenance - Periodic Practice to Maintain Student Mastery of Skills

Purpose: to provide periodic student practice activities and teacher directed review of this skill after students have mastered it.

1. Problem of the Day

Materials

- Multiplication problems
- Paper/pencil

Description:
The teacher will present a problem of the day verbally and by displaying the items in a designated area. Students will solve using base ten materials and place value mats. This should initially be done each day, then 2 times/we ek, wee $k l y, 6 i$ wee $k l y$, and then intermittently.
2. $\mathcal{T}$ ransition $\mathcal{F i l l e}$ r/Rapid Fire $\mathcal{V}$ erbal Revie $w$

Materials:

* Dependent on routine

Description:
While lining up or during other transitional times, the teacher will ask students to say/show the steps of the DRAW strategy (e.g. "O kay, while we are waiting to be called to the assembly, let's review what DRAW stands for (cover poster/display with mnemonic Doug, what does the $\mathcal{D}$ stand for? Right. Good. I ason, what about the R? $\mathcal{T a m m y}$, what about the $\mathcal{A}$, and $\mathcal{F e}$ licia the $\mathcal{W}$ ?) Continue to askstudents steps, reinforcing accurate and automatic responding.

