



California Partnership for Achieving Student Success

Algebra I California Content Standards

Standards Deconstruction Project

2005-2006

Version 1.2

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A note to the reader: This project was coordinated and funded by the California Partnership for Achieving Student Success (Cal-PASS). Cal-PASS is a data sharing system linking all segments of education. Its purpose is to improve student transition and success from one educational segment to the next.

Cal-PASS is unique in that it is the only data collection system that spans and links student performance and course-taking behavior throughout the education system—K–12, community college, and university levels. Data are collected from multiple local and state sources and shared, within regions, with faculty, researchers, and educational administrators to use in identifying both barriers to successful transitions and strategies that are working for students. These data are then used regionally by discipline-specific faculty groups, called “Intersegmental Councils,” to better align curriculum.

This Algebra 1 deconstruction project was initiated by the faculty serving on the math intersegmental councils after reviewing data on student transition. A deconstruction process was devised by the participating faculty with suggestions from the San Bernardino County Unified School District math faculty (Chuck Schindler and Carol Cronk) and included adaptations of the work of Dr. Richard Stiggins of the Assessment Training Institute and Bloom’s *Taxonomy of Educational Objectives* (Bloom, B. S., 1984, Boston: Allyn and Bacon). The following document represents a comprehensive review by K–16 faculty to deconstruct and align Algebra 1 (Elementary Algebra) standards.

In order to continue the collaboration on these standards, thus improving on the current work, we invite and encourage the reader to provide feedback to us. Please contact Dr. Shelly Valdez at: svaldez@calpass.org.

Table of Contents

California Content Standards 1-76

Standards 1–23 are presented in numerical order; each includes the following:

- Standard as written in the California DOE Algebra I Content Standards
- Deconstructed standard
- Prior knowledge needed by the student to begin learning the standard
- New knowledge student will need in order to achieve mastery of the standard
- Categorization of educational outcomes
- Necessary physical skills required
- Assessable result of the standard

Assessment Items

- Computational and procedural skills
- Conceptual understanding
- Problem solving/application

Appendix #1 78

List of Algebra I California Content Standards

Appendix #2 80

Developing Learning Targets for Algebra Standards
(Deconstruction instructions)

Appendix #3 85

Categorization of Educational Outcomes
(Explanation and instructions)

Appendix #4 87

Exemplar Practices for Standards 1–23

Standard #1

Standard Set 1.0

1.0 Students identify and use the arithmetic properties of subsets and integers and rational, irrational, and real numbers, including closure properties for the four basic arithmetic operations where applicable.

1.1 Students use properties of numbers to demonstrate whether assertions are true or false.

Deconstructed Standard

1. Students identify arithmetic properties of subsets of the real number system including closure for the four basic operations.
2. Students use arithmetic properties of subsets of the real number system including closure for the four basic operations.
3. Students use properties of numbers to demonstrate whether assertions are true or false.

Prior Knowledge Necessary

Students should:

- know the subsets of the real numbers system
- know how to use the commutative property
- know how to use the associative property
- know how to use the distributive property
- have been introduced to the concept of the addition property of equality
- have been introduced to the concept of the multiplication property of equality
- have been introduced to the concept of additive inverses
- have been introduced to the concept of multiplicative inverses

New Knowledge

Students will need to learn:

- how to apply arithmetic properties of the real number system when simplifying algebraic expressions
- how to use the properties to justify each step in the simplification process
- how to apply arithmetic properties of the real number system when solving algebraic equations
- how to use the properties to justify each step in the solution process
- how to identify when a property of a subset of the real numbers has been applied
- how to identify whether or not a property of a subset of the real number system has been properly applied
- the property of closure

Categorization of Educational Outcomes

Competence Level: Application

1. Students will identify arithmetic properties of subsets of the real number system including closure for the four basic operations.
2. Students use arithmetic properties of subsets of the real number system including closure for the four basic operations.
3. Students use properties of numbers to demonstrate whether assertions are true or false.(Justify)

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will provide examples and counterexamples to support or disprove assertions about arithmetic properties of subsets of the real number system.
2. Students will use arithmetic properties of subsets of the real number system to justify simplification of algebraic expressions.
3. Students will use arithmetic properties of subsets of the real number system to justify steps in solving algebraic equations.

Standard #1 Model Assessment Items

(Much of this standard is embedded in problems that are parts of other standards. Some of the examples below are problems that are from other standards that also include components of this standard.)

Computational and Procedural Skills

1. State the error made in the following distribution. Then complete the distribution correctly.

$$-4(x+2) = -4x+2$$

2. Solve the following equation and state the properties you used in each step.

$$3(x-2)-(x+5) = -22$$

3. Problem from the Los Angeles County Office of Education: Mathematics (National Center to Improve Tools of Education):

Which of the following sets of numbers are not closed under addition?

- a. the set of real numbers
- b. the set of irrational numbers
- c. the set of rational numbers
- d. the set of positive integers

Conceptual Understanding

1. Problem from the *Mathematics Framework for California Public Schools*:

Prove or give a counterexample:

The average of two rational numbers is a rational number.

2. Prove or give a counterexample:

$$x+x=x^2 \text{ for all real numbers } x.$$

Problem Solving/Application

1. The sum of three consecutive even integers is -66 . Find the three integers.

Standard #2

Standard Set 2.0

Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.

Deconstructed Standard

1. Students understand the operation of taking the opposite.
2. Students use the operation of taking the opposite.
3. Students understand the operation of finding the reciprocal.
4. Students use the operation of finding the reciprocal.
5. Students understand the operation of taking a root.
6. Students use the operation of taking a root.
7. Students understand the operation of raising to a fractional power.
8. Students use the operation of raising to a fractional power.
9. Students understand the rules of exponents.
10. Students use the operation of rules of exponents.

Prior Knowledge Necessary

Students should know:

- how to complete computations with rational numbers
- what an opposite is
- what a reciprocal is
- how to take the square root of a perfect square
- how to estimate the value of the square root of a non-perfect square
- how to simplify arithmetic expressions using the exponent rules
- how to simplify simple algebraic expressions using the exponent rules

New Knowledge

Students will need to:

- learn to use opposites in more complex rational number problems
- learn to use opposites with variable terms $(-x) + (x) = 0$
- learn to use the reciprocal as a tool for solving any multiplication or division component of an equation

<u>Old Method</u>	<u>New Method</u>
$3x = -9$	$3x = -9$
$\frac{3x}{3} = \frac{-9}{3}$	$\frac{1}{3}(3x) = \frac{1}{3}(-9)$
$x = -3$	$x = -3$

- extend knowledge of square roots from perfect squares and square roots that are between two integers to a deeper understanding of the concept of square roots and the rational number system
- learn to simplify square roots
- be introduced to fractional roots other than square roots

- learn to use all of the rules of exponents
- learn to simplify more complex computations with monomials including more complex simplification of rational expressions

Categorization of Educational Outcomes

Competence Level: Comprehension and Application

1. Students will understand the operations of:
 - a. taking the opposite
 - b. finding the reciprocal
 - c. taking a root
 - d. raising to a fractional power
 - e. the rules of exponents
2. Students will use the operations of:
 - a. taking the opposite
 - b. finding the reciprocal
 - c. taking a root
 - d. raising to a fractional power
 - e. the rules of exponents

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will simplify arithmetic expressions by using the rules of exponents and the operations of taking a root and raising to a fractional power.
2. Students will simplify algebraic expressions by using the rules of exponents and the operations of taking an opposite, finding a reciprocal, taking a root, and raising to a fractional power.
3. Students will solve algebraic equations by using the rules of exponents and the operations of taking an opposite, finding a reciprocal, taking a root, and raising to a fractional power.

Standard #2 Model Assessment Items

(Most of the problems students will complete in relation to this standard will be embedded in problems that are parts of other standards. For example, problems that involve solving multi-step linear equations, quadratics, and rational expressions all require the components of this standard to complete.)

Computational and Procedural Skills

1. Released item from CAHSEE:

If $x = -7$, then $-x =$

- a. -7
- b. $-1/7$
- c. $1/7$
- d. 7

If $x = -7$, then $-x =$

A -7 B $-\frac{1}{7}$ C $\frac{1}{7}$ D 7

2. Released item from the *Algebra I CST*:

$$\sqrt{16} + \sqrt[3]{8} =$$

A 4 B 6 C 9 D 10

3. Problem from the *Mathematics Framework for California Public Schools*:

Write as a power of x : $\frac{\sqrt{x}}{x^3\sqrt{x}}$

Conceptual Understanding

1. Problem from the *Mathematics Framework for California Public Schools*:

What must be true about the real number x if $x = \sqrt{x^2}$

2. For what value of x is the following true: $x = -x$

Problem Solving/Application

1. Released item from CAHSEE:

The perimeter, P , of a square may be found

by using the formula $\left(\frac{1}{4}\right)P = \sqrt{A}$, where A is

the area of the square. What is the perimeter

of the square with an area of 36 square

inches?

A 9 inches C 24 inches

B 12 inches D 72 inches

2. Problem from the *Mathematics Framework for California Public Schools*:

I start with a number and apply a four-step process: I add 13, multiply by 2, take the square root and then take the reciprocal. The result is $\frac{1}{4}$. What number did I start with?

Standard #3

Standard 3.0

Students solve equations and inequalities involving absolute values.

Deconstructed Standard

1. Students use the real number line to solve equations involving absolute value.
2. Students solve equations involving absolute value numerically.
3. Students solve equations involving absolute value analytically.
4. Students use the real number line to solve inequalities involving absolute value.
5. Students solve inequalities involving absolute value numerically.
6. Students solve inequalities involving absolute value analytically.

Prior Knowledge Necessary

Students should know how to:

- solve algebraic equations
- solve algebraic inequalities
- find the absolute value of a number
- simplify algebraic expressions
- evaluate absolute value expressions for a given value of the variable
- create a t-table for an equation
- plot points on the real number line
- graph solutions to an inequality on the real number line
- interpret the absolute value of a number as a distance from zero on a number line
- interpret the distance between two points on the number line as the absolute value of the difference of the points (the distance between x and a is $|x - a|$)
- create a t-table for an absolute value equation
- represent solution sets using inequalities and/or interval notation
- convert solution sets written in interval notation to inequality notation and vice versa
- graph solution sets on the real number line given either interval notation or inequality notation

New Knowledge

Students will need to learn to:

- identify absolute value equations that have no solution (e.g., $|x - 3| = -9$)
- identify absolute value inequalities that have no solution (e.g., $|x + 2| < 0$)
- identify an absolute value equation as the distance between two points on the real number line and plot the appropriate points accordingly
- identify an absolute value inequality as a distance on the real number line and graph the appropriate regions accordingly
- create a guess and check table to solve an absolute value equation
- create a guess and check table to solve an absolute value inequality
- solve an absolute value equation by removing absolute value bars and solving the resulting equation(s)

- solve an absolute value inequality by removing absolute value bars and solving the resulting inequalities
- represent the solution set to an absolute value inequality by graphing the solution set on the real number line
- represent the solution set to an absolute value inequality by writing an appropriate inequality (union or intersection)
- represent the solution set to an absolute value inequality with the appropriate interval notation (union or intersection)

Categorization of Educational Outcomes

Competence Level: Application

1. Students will interpret distance problems in terms of absolute value.
2. Students will classify solution sets to absolute value equations/inequalities as the union or intersection of two sets.
3. Students will apply previously acquired knowledge and skills to solving absolute value equations/inequalities analytically, numerically, and/or graphically.
4. Students will translate computational skills required to solve absolute value equations and inequalities to real world application problems.

Necessary New Physical Skills

1. Use of ruler/straight edge.
2. Use of graph paper to graph equations and inequalities.
3. Ability to sketch a graph.

Assessable Result of the Standard

1. The student will represent the solution set to an absolute value equation by plotting points on the real number line.
2. The student will represent the solution set to an absolute value equation as the union or intersection of two sets in interval notation.
3. The student will represent the solution set to an absolute value equation as a compound inequality or as the union of two disjoint inequalities.
4. The student will represent the solution set to an absolute value inequality as a graph on the real number line.
5. The student will represent the solution set to an absolute value inequality as the union or intersection of two sets in interval notation.
6. The student will represent the solution set to an absolute value inequality as a compound inequality or as the union of two disjoint inequalities

Standard #3 Model Assessment Items

Computational and Procedural Skills

1. Solve $|2x - 3| = 2$ analytically.
2. Solve $4|x + 2| - 3 > 13$ analytically and write your result as a compound inequality if possible.
3. Solve $\left|\frac{x - 2}{3}\right| \leq 2$ analytically and write your result as a compound inequality if possible.
4. Solve $|x - 3| = 2$ by plotting the appropriate distance from 3 on the real number line.
5. Solve $|x + 2| \leq 12$ by graphing the appropriate distance(s) on the real number line.
6. Solve $3 - |x + 4| \leq 1$ analytically and write the solution set in interval notation.
7. Solve $|2x - 3| > 2$ numerically (by creating a guess and check table) and graph the solution set on the real number line.

Conceptual Understanding

1. Write a sentence interpreting $|x - 3| = 5$ as a distance on the real number line and graph the solution set on the real number line.
2. Write a sentence interpreting $|x + 4| \leq 3$ as a distance on the real number line and graph the solution set on the real number line.
3. Write a sentence interpreting $|x - 7| > 4$ as a distance on the real number line and graph the solution set on the real number line.

Problem Solving/Application

1. Often students will try to solve an inequality like an equation. For example, a student might try to solve $1 < \frac{3}{x}$ by multiplying both sides by x to get $x < 3$. However $x = -1$ satisfies $x < 3$ but does not satisfy $1 < \frac{3}{x}$. What's wrong? Explain why multiplying both sides of the original inequality by x doesn't work.
2. Recall that $|a - b|$ is the distance between a and b on the real number line. For any number x , what do $|x - 1|$ and $|x - 3|$ represent? Use this interpretation to solve the inequality $|x - 1| < |x - 3|$ geometrically.

Standard #4

Standard Set 4.0

Students simplify expressions prior to solving linear equations and inequalities in one variable, such as $3(2x - 5) + 4(x - 2) = 12$.

Deconstructed Standard

1. Students simplify expressions prior to solving linear equations in one variable.
2. Students simplify expressions prior to solving linear inequalities in one variable.

Prior Knowledge Necessary

Students should know how to:

- add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals)
- use algebraic terminology (e.g., variable, equation, term, coefficient)
- apply algebraic order of operations and the commutative, associative, and distributive properties to evaluate expressions

New Knowledge

Students will need to learn to:

- combine like terms to simplify algebraic expressions
- use the distributive property to simplify algebraic expressions
- master multiplication and division of monomials
- simplify more complicated algebraic expressions requiring more than one operation

Categorization of Educational Outcomes

Competence Level: Application

1. Students will use methods they have learned to combine like terms in an expression.
2. Students will demonstrate their ability to distribute a monomial through a polynomial.
3. Students will demonstrate their ability to distribute a negative value through a polynomial.
4. Students will demonstrate their ability to distribute a fractional expression through a polynomial.
5. Students will be able to combine several skills in order to simplify an expression.

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will produce a simplified expression containing no like terms.

Standard #4 Model Assessment Items

Computational and Procedural Skills

1. Simplify the following expressions:

a. $a+2-b-8$

b. $-\frac{5}{2}x-6+\frac{1}{4}x-5$

c. $5(3x-4)$

d. $-3(2x-5)$

e. $-2(3x-1)-2(5x-4)$

f. $6\frac{1}{3}x-\frac{1}{2}y+4\frac{1}{8}x+\frac{3}{4}y$

g. $4-(3x-2y-1)-5(x-y+5)$

h. $2x(3x^2-2x+1)$

Standard #5

Standard Set 5.0

Students solve multi-step problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.

Deconstructed Standard

1. Students will solve multi-step linear equations in one variable.
2. Students will solve multi-step linear inequalities in one variable.
3. Students will set up word problems involving linear equations.
4. Students will solve word problems involving linear equations.
5. Students will set up word problems involving linear inequalities.
6. Students will solve word problems involving linear inequalities.
7. Students will be able to identify the property being used at each step when solving a multi-step linear equation.
8. Students will be able to identify the property being used at each step when solving a multi-step linear inequality.

Prior Knowledge Necessary

Students should have learned:

- to simplify algebraic expressions using the distributive property
- to combine like terms
- to correctly identify and use inequality symbols
- to add, subtract, multiply, and divide rational numbers
- to translate verbal phrases into mathematical expressions
- mathematical properties such as associative property, commutative property, and the distributive property
- to solve one-step algebraic equations
- the order of operations
- the perimeter formula for a square and a rectangle

New Knowledge

Students will need to learn to:

- isolate the variable in multi-step linear equations
- isolate the variable in multi-step linear inequalities
- apply the multiplicative property of inequalities
- justify each step when solving a linear equation or inequality
- translate word problems into multi-step linear equations or inequalities
- interpret the results from a multi-step word problem
- apply standard formulas such as distance, rate, and time
- represent consecutive integer problems using algebraic expressions

Categorization of Educational Outcomes

Competence Level: Application

1. Students will use methods they have learned to solve a multi-step linear equation.
2. Students will demonstrate their ability to solve a multi-step linear inequality.
3. Students will demonstrate their ability to set up and solve a variety of multi-step word problems requiring the solving a linear equation or inequality to obtain the answer.
4. Students will demonstrate their knowledge by being able to correctly justify each step when solving a multi-step linear equation or inequality.

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will find solutions to multi-step equations.
2. Students will find solutions to multi-step inequalities.
3. Students will find solutions to word problems.

Standard #5 Model Assessment Items

1. Simplify the following expressions:

a. $2x+3=15$

b. $\frac{1}{4}x-3=-8$

c. $-4(m+6)=-36$

d. $-3(2x-5)$

e. $\frac{x-8}{4}-3=-2$

f. $5x+2(1-x)=2(2x-1)$

g. $-3x<9$

h. $4x-7\leq 5x+3$

i. $5(x-1)>\frac{5}{4}x$

j. $8-2y\geq 4-y$

k. Solve $\frac{x-8}{4}-3=-2$ and justify each step.

l. The length of a rectangle is six less than twice the width. Its perimeter is 36 inches. Find the dimensions of the rectangle.

m. The greater of two consecutive integers is 15 more than twice the smaller. Find the integers.

n. The sum of 32 and twice a number is 118. Find the number.

o. Jan hiked up a hill at 4 mi/hr and back down at 6 mi/hr. Her total hiking time was 3 hours. How long did the trip up the hill take her?

p. The sum of two consecutive positive integers is at most 18. What are the integers?

q. Seven times a number decreased by 4 times the number is less than 30. What is the number?

Standard #6

Standard Set 6.0

Students graph a linear equation and compute the x - and y -intercepts (e.g., $2x + 6y = 4$). They are also able to sketch the region defined by a linear inequality (e.g., they sketch the region defined by: $2x + 6y < 4$).

Deconstructed Standard

1. Students graph linear equations.
2. Students compute x -intercepts.
3. Students compute y -intercepts.
4. Students use intercepts to graph linear equations.
5. Students sketch the region defined by a linear inequality.

Prior Knowledge Necessary

Students should know how to:

- perform arithmetic computations with rational numbers
- graph ordered pairs
- compute slope from the graph of a line
- compute slope when given two points
- recognize slope as a rate of change of y in relation to x
- graph a linear equation using a t -chart
- evaluate a linear equation for a given x or y value
- solve one-variable linear inequalities
- graph the solution set for a one-variable linear inequality
- verify that any element in the solution set of a one-variable inequality satisfies the original inequality.

New Knowledge

Students will need to learn to:

- plot the y -intercept (given the slope/intercept form of a line, $y = mx + b$), and then use the slope to find a second point in order to complete the graph of the line
- identify the graphical representation of $(a, 0)$ as the x -intercept, and $(0, b)$ as the y -intercept
- compute the x -intercept and y -intercept given a linear equation
- identify that the linear equation implied by the linear inequality forms a boundary for the solution set and that this boundary may or may not be included in the final graph
- interpret the inequality symbol to determine whether or not the boundary is solid or dashed
- identify and shade the region of the graph that contains the solutions to the inequality
- recognize that linear inequalities have multiple ordered-pair solutions

Categorization of Educational Outcomes

Competence Level: Application

1. Students will use methods they have learned to graph lines, solve inequalities, and to locate and/or identify the x - and y -intercepts for a given equation or graph.
2. Students will demonstrate their ability to find and use x - and y -intercepts in the context of graphing.
3. Students will calculate x - and y -intercepts.
4. Students will solve inequalities in two variables.
5. Students will use information they have learned to graph lines, solve inequalities, and find x - and y -intercepts.
6. Students will show that they know the correct interpretation of the boundary line for the solution of an inequality by appropriately making the boundary solid or dashed.

Necessary New Physical Skills

1. Use of a ruler

Assessable Result of the Standard

1. Students will produce the graph of a line.
2. Students will produce the ordered pairs representing the x - and y -intercepts.
3. Students will produce a bounded and shaded region of the x - y plane representing the solution set of a linear inequality in two variables.

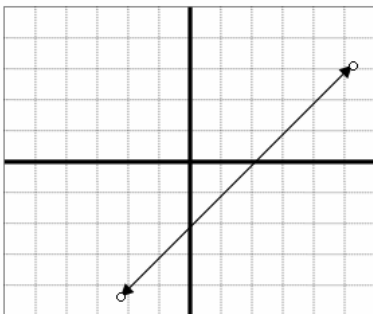
Standard #6 Model Assessment Items

Computational and Procedural Skills

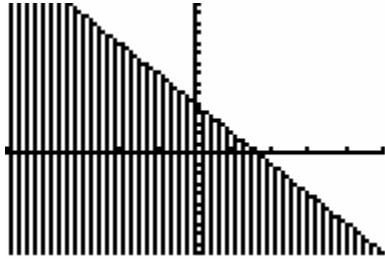
- Find the x - and y -intercepts for the line defined by the following equation:
 $2x + 3y = 9$
- Use the x - and y -intercepts to graph the line given by the equation above:
 $2x + 3y = 9$
- Graph the following lines using the method of your choice. Identify and label the x - and y -intercepts for each graph if they exist:
 - $3x - 5y = 10$
 - $y = \frac{-2}{3}x + 4$
 - $y = 2$
 - $x = 3.5$
 - $2x + 4y = 3$
 - $\frac{1}{2}x - \frac{3}{4}y = 2$
- Graph the solution set for the following inequalities:
 - $2x - 3y < 6$
 - $y \geq \frac{-3}{4}x + 2$
 - $\frac{1}{2}x - \frac{2}{3}y \leq \frac{5}{6}$

Conceptual Understanding

- Sketch the graph of a line that has no x -intercept.
- Identify the x - and y -intercepts from the graph of the given line.



- Can a line have more than one x -intercept? Explain your answer using a diagram.
- The solution to an inequality has been graphed correctly below. Insert the correct inequality symbol in the inequality below to match the graph of the solution.
(Everything else about the inequality is correct—it just needs the correct symbol).



$$y \boxed{\phantom{<}} 3x + 5$$

Insert correct symbol in box.

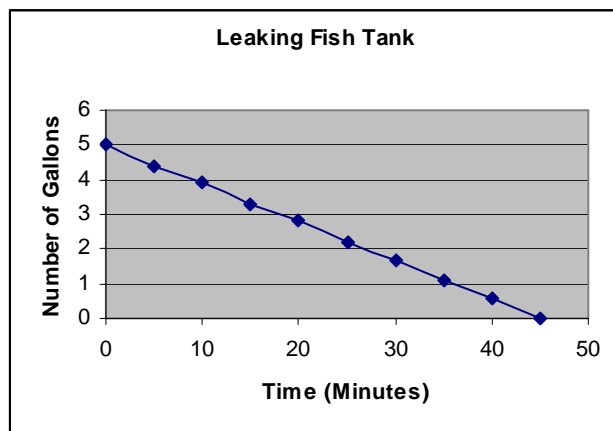
- When is it advantageous to use the x - and y -intercepts to graph the equation of a line? When would it perhaps be easier or better to use another graphing method? Give an example to illustrate your answers to both of these questions.

Problem Solving/Application

- The graph displayed below is the graph of the following equation: $y = \left(\frac{-1}{9}\right)x + 5$,

where x represents the amount of time that has passed since a 5 gallon fish tank sprung a leak, and y represents the number of gallons of water in the tank after the leak.

- What is the significance of the x -intercept in this situation? What information is given to us by this point?
- What is the significance of the y -intercept in this situation? What information is given to us by this point?



- The cost of a trash pickup service is given by the following formula:
 $y = 1.50x + 11$, where x represents the number of bags of trash the company picks up, and y represents the total cost to the customer for picking up the trash.

- a. What is the y -intercept for this equation?
- b. What is the significance of the y -intercept in this situation? What does it tell us about this trash pickup service?
- c. Draw a sketch of the graph which represents this trash pickup service.

Standard #7

Standard Set 7.0

Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations using the point-slope formula.

Deconstructed Standard

1. Given a point, students will evaluate a linear equation in two variables for the given values of the variables to determine whether or not the point lies on the line.
2. Students will derive the equation of a line given a graph of the line.
3. Students will derive the equation of a line given two points on the line.
4. Students will derive the equation of a line given the slope of the line and one point on the line.
5. Students will derive the equation of a line given the slope of the line and the y-intercept.
6. Students will derive the equation of a line given data in table form (consisting of two or more input-output data points).

Prior Knowledge Necessary

Students should know how to:

- evaluate an algebraic expression in two variables for the given values of the variables
- simplify algebraic expressions—particularly by using the distributive property
- graph linear equations
- define the slope of a line as the ratio of the change in y with respect to the change in x

New Knowledge

Students will need to learn how to:

- verify that an order pair of values satisfies the equation of a line in two variables
- identify the y -intercept given the graph of a line
- determine the slope of a line given its graph
- calculate the slope of a line given two points on the line
- write an equation of the line in standard form, point-slope form, and/or slope-intercept form, given the slope of a line and one point on the line
- write the equation of the line in standard form, point-slope form, and/or slope-intercept form, given the slope of a line and the y -intercept
- write the equation of the line in standard form, point-slope form, and/or slope-intercept form, given two points on the line
- identify the x variable, identify the y variable, and write the equation of the line in standard form, point-slope form, and/or slope-intercept form, given a table of linear data points as ordered pairs of numbers
- write the equation of the line in standard form, point-slope form, and/or slope-intercept form, given the graph of a line

Categorization of Educational Outcomes

Competence Level: Application

1. Students will interpret the slope of a line as a rate of change.
2. Students will interpret the y-intercept as the initial value in application problems involving linear data.
3. Given particular data values, students will choose the appropriate equation form to write an equation of a line.
4. Students will apply previously acquired knowledge and skills to writing the equation of a line.
5. Students will demonstrate whether or not a point lies on a line.

Necessary New Physical Skills

1. Use of a ruler/straight edge.
2. Use of graph paper to graph equations.
3. Ability to sketch a graph.
4. Ability to operate a calculator.

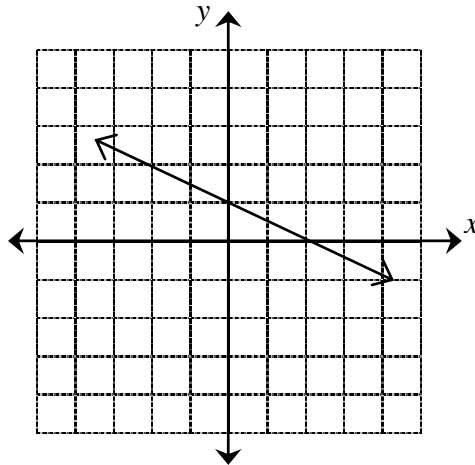
Assessable Result of the Standard

1. Students will produce the equation of a line in standard form, point-slope form, and/or slope-intercept form.
2. Students will produce a linear model (equation) representing data from an application problem.

Standard #7 Model Assessment Items

Computational and Procedural Skills

1. Given the equation of a line, $y = \frac{3}{2}x - 1$, determine whether or not the following points lie on the line: $(4, 7)$, $(-6, -10)$, and $(\frac{1}{3}, -\frac{1}{2})$.
2. Write the equation of the line with slope $\frac{2}{3}$ and y-intercept of -2 and choose the appropriate form of the line (either point-slope or slope-intercept).
3. Write the equation of the line with a slope of 4 and passing through the point $(-2, 5)$ and choose the appropriate form of the line (either point-slope or slope-intercept).
4. Write the equation of the line passing through the points $(-3, 7)$ and $(4, -7)$, and be sure to choose the appropriate form of the line (either point-slope or slope-intercept).
5. Write the equation of the line graphed below.



Conceptual Understanding

Consider the table of values below:

Episode Number	Number of Survivors
1	25
2	23
3	21

- a. Identify the input and output variables.

- b. Determine whether or not the data is linear.
- c. If the data is linear, find and interpret the slope.

Problem Solving/Application

A taxi driver charges a \$2 flag-drop fee plus \$3 for each mile traveled.

- a. Identify the input and output variables.
- b. Make a table for inputs 0, 1, and 2.
- c. Identify and interpret the meaning of the slope.
- d. Identify and interpret the meaning of the vertical axis intercept.
- e. Write an equation describing the output as a function of the input.
- f. Use your equation to determine the cost of a 12-mile taxi ride.

Standard #8

Standard Set

Students understand the concepts of parallel lines and perpendicular lines and how those slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.

Deconstructed Standard

1. Students understand concepts of parallel lines.
2. Students understand concepts of perpendicular lines.
3. Students understand the relationship between parallel and perpendicular lines.
4. Students determine the equation of a line perpendicular to a given line that passes through a given point.

Prior Knowledge Necessary

Students should know how to:

- identify the slope and y-intercept from the equation of a line
- identify the slope and y-intercept from the graph of a line
- convert the equation of a line into slope-intercept form
- determine the opposite and reciprocal of a rational number
- determine the equation of a line given a point and a slope

New Knowledge

Students will need to learn how to:

- identify parallel lines as having the same slope and different y-intercepts graphically
- identify parallel lines as having the same slope and different y-intercepts algebraically
- identify perpendicular lines graphically by intersecting at 90° angles
- identify perpendicular lines algebraically as having slopes whose product is -1
- understand the relationship between the slopes of parallel lines
- determine the equation of a line parallel to a given line and passing through a point not on the line
- determine the equation of line perpendicular to a given line and passing through a point not on the line

Categorization of Educational Outcomes

Competence Level: Knowledge

1. Students will identify pairs of lines as parallel, perpendicular, or neither by their slopes.
2. Given the graph of a line, students will construct parallel and perpendicular lines.

Competence Level: Comprehension

1. Students will compare and contrast the relationships between parallel and perpendicular lines.

Competence Level: Application

1. Given the equation of a line and a point not on the line, a student will be able to calculate the equation of a parallel line.
2. Given the equation of a line and a point not on the line, a student will be able to calculate the equation of a perpendicular line.

Necessary New Physical Skills

1. Use of a ruler.

Assessable Result of the Standard

1. Students will graph parallel lines.
2. Students will graph perpendicular lines.
3. Students will calculate the equation of a line parallel to a given line and passing through a given point not on the line.
4. Students will calculate the equation of a line perpendicular to a given line and passing through a given point not on the line.

Standard #8 Model Assessment Items

Computational and Procedural Skills

1. Determine the equation of a line parallel to a given line and passing through a point not on the line:
Find an equation of a line, in slope-intercept form, through point $P(3,-2)$ that is parallel to $3y + 5x = 7$.
2. Determine the equation of a line perpendicular to a given line and passing through a point not on the line:
Find an equation of a line, in slope-intercept form, through point $P(-12,4)$ that is perpendicular to $y - 6x = 3$.

Conceptual Understanding

3. Recognize parallel lines as having the same slope and different y-intercepts graphically:
Graph: $y = 2x + 4$ and $y - 2x = 1$ on the same rectangular coordinate system. Does it appear the two lines are parallel? Explain.
4. Recognize parallel lines as having the same slope and different y-intercepts algebraically:
Algebraically, demonstrate the two lines whose equations are given by: $2y - 4x = 9$ and $y = 2x - 5$ are parallel.
5. Recognize perpendicular lines algebraically by intersecting at 90° angles:
Graph: $y + x = 5$ and $y - x = 0$ on the same rectangular coordinate system. Does it appear the two lines are perpendicular? Explain.
6. Recognize perpendicular lines algebraically as having slopes whose product is -1 :
Algebraically, demonstrate the two lines whose equations are given by: $4y - 3x = 7$ and $3y + 4x = 15$ are perpendicular.

Standard #9

Standard Set 9.0

Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.

Deconstructed Standard

1. Students solve systems of two equations in two variables algebraically.
2. Students interpret solutions to systems of two equations in two variables graphically.
3. Students solve systems of two inequalities in two variables by sketching the solution set.

Prior Knowledge Necessary

Students should know how to:

- identify points on a Cartesian coordinate system
- solve linear equations in one variable
- solve linear inequalities in one variable
- graph lines to solve systems of linear equations
- graph linear equations in two variables
- graph linear inequalities in two variables
- isolate a variable in an equation of two variables

New Knowledge

Students will need to learn how to:

- solve systems of two equations in two variables using substitution
- solve systems of two equations in two variables using elimination/addition method
- interpret the solution as representing parallel lines, the intersection of two lines or the same line
- identify the intersection of two shaded areas created by the graphs of linear inequalities in two variables as the solution to the system

Categorization of Educational Outcomes

Competence Level: Application

1. Students will use appropriate methods to solve systems of two equations in two variables.
2. Students will interpret the solution of a system graphically.
3. Students will determine the solutions set from the graph of a system of linear inequalities in two variables.

Necessary New Physical Skills

1. Use of ruler.

Assessable Result of the Standard

1. Students will find solutions to systems of equations with their appropriate graphical representation.
2. Students will find shaded-area solutions for systems of linear inequalities.

Standard #9 Model Assessment Items

Computational and Procedural Skills

1. Solve systems of two equations in two variables using substitution:

Solve the system: $\begin{cases} 2x + y = 6 \\ -3x - 2y = -15 \end{cases}$ by the substitution method.

2. Solve systems of two equations in two variables using elimination:

Solve the system: $\begin{cases} 3x + 5y = 18 \\ x - 2y = -5 \end{cases}$ by the elimination method.

3. Identify the intersection of two shaded areas created by the graphs of linear inequalities in two variables as the solution to the system:

Solve the system of inequalities $\begin{cases} x + y < 7 \\ 2x - 3y \geq 12 \end{cases}$ graphically.

Conceptual Understanding

1. Interpret the solution as representing the intersection of two lines or the same line:
Suppose a system of two linear equations in two variables has a solution, (3,5). What would you expect the graph to look like?

Suppose a system of two linear equations in two variables has infinite solutions in the form (x,y). When graphing these two equations on the same Cartesian coordinate system, what would you expect the graph of the two lines to look like?

Solve the system: $\begin{cases} y - 3x = -3 \\ 3x + 2y = 12 \end{cases}$ by graphical method.

Solve the system: $\begin{cases} y = 3x + 6 \\ -6x + 2y = 12 \end{cases}$ by graphical method.

Standard #10

Standard Set 10.0

Students add, subtract, multiply, and divide monomials and polynomials. Students solve multi-step problems, including word problems, by using these techniques.

Deconstructed Standard

1. Students add monomials and polynomials.
2. Students subtract monomials and polynomials.
3. Students multiply monomials and polynomials.
4. Students divide monomials and polynomials.
5. Students solve multi-step problems involving monomials and polynomials.
6. Students solve word problems.

Prior Knowledge Necessary

Students should know how to:

- combine like terms
- use the distributive property to simplify expressions
- add, subtract, multiply, and divide rational numbers
- translate multi-step application problems to algebraic expressions
- solve multi-step problems

New Knowledge

Students will need to learn how to:

- collect like terms to add and subtract polynomials
- use the distributive property to multiply polynomials
- apply the appropriate exponential rules to simplify algebraic expressions
- divide polynomials by binomials using long division
- translate and solve multi-step word problems involving polynomial arithmetic

Categorization of Educational Outcomes

Competence Level: Application

1. Apply the rules of arithmetic to monomials and polynomials.
2. Demonstrate the ability to interpret word problems into algebraic symbols and solve them using the arithmetic of polynomials.
3. Interpret and solve word problem.

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will simplify polynomials.
2. Students will find solutions to a variety of word problems requiring the arithmetic of polynomials.

3. Students will find solutions to a variety of multi-step problems requiring the arithmetic of polynomials.

Standard #10 Model Assessment Items

Computational and Procedural Skills

1. The process of adding, subtracting, multiplying and dividing monomials and polynomials:

a. Combine $2x^3y^2 + 3x^3y^2 - 7x^3y^2$.

b. Multiply and simplify if possible: $4x^5y^4\left(\frac{3}{8}x^2y\right)$

- c. Multiply and simplify if possible:

1. $3x^2(7x - 2)$

2. $(3x + 7)(2x - 6)$

3. $(x - 7)(x + 7)$

4. $\frac{1}{2}xy(2y - 4x^2)$

5. $(x - y)(x^2 + xy + y^2)$

- d. Divide and simplify if possible:

1. $\frac{12x^3y^2}{4x^5y}$

2. $\frac{18x^2 + 12x + 6}{6x}$

3. $\frac{2x^3 - 3x^2 + 7}{x - 2}$

Problem Solving/Application

1. The process for setting up and solving multi-step problems including work:
- a. Jane can mow a lawn in 2 hours. Tom can mow the same lawn in 3 hours. How long would it take both of them, working together, to mow the lawn?
- b. The sum of a number and 20 times its reciprocal is -9 . Find the number.

Standard #11

Standard Set 11.0

Students apply basic factoring techniques to second- and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.

Deconstructed Standard

1. Students apply basic factoring techniques to second-degree polynomials, i.e., factoring a quadratic expression into the product of two binomials, and factoring a quadratic expression using grouping.
2. Students apply basic factoring techniques to simple third-degree polynomials.
3. Students find a common factor for all terms in a polynomial.
4. Students recognize the difference of two squares.
5. Students recognize perfect squares of binomials.

Prior Knowledge Necessary

Students should know how to:

- identify the greatest common factor (GCF) of two or more numbers
- identify and apply the distributive property of multiplication
- factor a simple linear expression
- multiply monomials
- multiply binomials
- multiply monomials with binomials
- prime factor numbers
- prime factor monomial terms
- identify second-degree polynomials
- identify third-degree polynomials

New Knowledge

Students will need to learn how to:

- identify the greatest common factor (GCF) of a polynomial
- factor a polynomial using the GCF of a polynomial
- factor a second-degree polynomial as the product of two binomials, i.e., Reverse FOIL method, Master Product method or AC method
- factor a second-degree polynomial using grouping
- recognize that a polynomial (binomial) is actually the difference of two square terms
- use multiplication of binomial squares, i.e., $(x + 2)^2$, to recognize that a polynomial is a perfect square of binomials
- use GCF and difference of squares, and perfect squares of binomials to factor simple third-degree polynomials

Categorization of Educational Outcomes

Competence Level: Knowledge

1. Students will identify the GCF of a polynomial.
2. Students will identify a second-degree polynomial as the product of two binomials.
3. Students will identify a polynomial as the difference of two square monomial terms.
4. Students will identify a polynomial as the square of a binomial.

Competence Level: Application

1. Students will use methods of factoring the GCF from a polynomial.
2. Students will use the methods of grouping, Reverse FOIL, or other methods to factor a second-degree polynomial.
3. Students will demonstrate knowledge of the difference of two square monomials to factor a polynomial.
4. Students will demonstrate knowledge of perfect square binomials to factor a polynomial.
5. Students will use a combination of one or more of the above methods to factor simple third-degree polynomials.

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will generate factors of a polynomial.

Standard #11 Model Assessment Items

Computational and Procedural Skills

1. Factor:
 - a. $x^2 + 2x$
 - b. $x^2 + 6x + 7$
 - c. $m^2 - 25$
 - d. $4z^2 + 24z - 13$
 - e. $x^3 + 2x^2 - 3x$

Conceptual Understanding

1. How can you check your answer when you factor a polynomial?
2. Given the expression $5y(2x - 3) + 8(2x - 3)$, is the expression completely factored? If not, what is the next step in factoring this expression.
3. If you are asked to completely factor the polynomial $3x^2 + 9x - 12$, why would it be incorrect to give $(x - 1)(3x + 12)$ as your answer?

Problem Solving/Applications

1. Find a value of b so that $x^2 + bx + 25 = (x + 5)^2$.
2. Find a so that $ay^2 - 12y + 4 = (3y - 2)^2$.

Standard #12

Standard Set 12.0

Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.

Deconstructed Standard

1. Students apply basic factoring techniques to second-degree polynomials.
2. Students simplify rational expressions by identifying common factors in the numerator and denominator and reducing them.

Prior Knowledge Necessary

Students should know how to:

- perform the basic arithmetic operations with fractions
- factor polynomials of the form $ax^2 + bx + c$ for $a = 1$ and $a \neq 1$
- factor third-degree polynomials by factoring the greatest common factor to reduce the polynomial to a trinomial
- recognize and factor the difference of perfect squares
- recognize and factor trinomials as the square of a binomial
- simplify numeric fractions by reducing common factors in the numerator and denominator

New Knowledge

Students will need to learn how to:

- apply factoring techniques to the numerator and denominator of rational expressions
- identify common factors in the numerator and denominator of a rational expression
- reduce rational expressions by canceling common factors in the numerator and denominator

Categorization of Educational Outcomes

Competence Level: Knowledge

1. Students will be able to identify common factors in the numerator and denominator of a rational function.
2. Students will be able to recognize a common factor in the numerator and denominator as being equivalent to 1 and hence canceling them.
3. Students will be able to recognize when a rational expression has been reduced to its lowest terms.

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will reduce rational expressions to their lowest terms.

Standard #12 Model Assessment Items

Computational and Procedural Skills

1. Applying factoring techniques to the numerator and the denominator of rational expressions.

Simplify the rational expressions:

a. $\frac{x^3 - 9x}{x^4 - 81}$

b. $\frac{x^2 + 10x + 25}{x^2 - 25}$

2. Identifying common factors in the numerator and denominator of a rational expression as equivalent to one.

Simplify the rational expressions:

a. $\frac{(x-1)(x+2)}{(x+2)(x-1)}$

b. $\frac{x^2 - 1}{1 - x^2}$

3. Reducing rational expressions by canceling common factors in the numerator and the denominator.

Simplify the rational expressions:

a. $\frac{x^3 - 9x}{x^4 - 81}$

b. $\frac{x^2 + 10x + 25}{x^2 - 25}$

c. $\frac{4x^2 - 12x + 9}{4x - 6}$

d. $\frac{7x^3 + 14x^2 - x^2 - 2x}{49x^2 - 14x + 1}$

e. $\frac{16x^2 + 24x + 9}{4x^2 - x - 3}$

Standard #13

Standard Set 13.0

Students add, subtract, multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems by using these techniques.

Deconstructed Standard

1. Students add rational expressions.
2. Students subtract rational expressions.
3. Students multiply rational expressions.
4. Students divide rational expressions.
5. Students add rational functions.
6. Students subtract rational functions.
7. Students multiply rational functions.
8. Students divide rational functions.
9. Students solve computationally challenging problems involving rational expressions and functions.
10. Students solve conceptually challenging problems involving rational expressions and functions

Prior Knowledge Necessary

Students should know how to:

- identify the least common denominator (LCD) of two or more fractions
- simplify rational expressions
- add, subtract, multiply, and divide simple rational expressions not involving polynomials (i.e., fractions)
- identify when a rational expression is undefined
- factor second-degree and simple third-degree polynomials
- add, subtract, and multiply polynomials

New Knowledge

Students will need to learn how to:

- use the Fundamental Property of Rational Expressions to simplify rational expressions prior to operating on them
- multiply rational expressions, simplifying the resulting rational expression into lowest terms
- divide rational expressions (i.e., multiply by the reciprocal of the denominator), simplifying the resulting expression to its lowest terms
- add rational expressions, using the least LCD and simplify the resulting expression into lowest terms
- subtract rational expressions, using the LCD and simplify the resulting expression into lowest terms
- apply the above techniques/knowledge to rational functions/equations

Categorization of Educational Outcomes

Competence Level: Knowledge

1. Students will identify a rational expression and a rational equation/function.
2. Students will identify for what values a rational expression/function is undefined.

Competence Level: Application

1. Students will use methods of factoring polynomials to reduce rational expressions/functions to their lowest terms.
2. Students will use methods of factoring polynomials to identify the LCD of rational expressions and equations involving addition and subtraction.
3. Students will use methods of factoring polynomials in simplifying rational expressions/equations in multiplying or dividing rational expressions.

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will generate reduced expressions for operations involving rational expressions.
2. Students will generate solutions for rational functions/equations.
3. Students will identify when a rational expression or function is undefined.

Standard #13 Model Assessment Items

Computational and Procedural Skills

1. Write the rational expressions in lowest terms:

$$\frac{7m+14}{5m+10}, \frac{16x^2-9y^2}{12x-9y}$$

2. Multiply. Write the answer in lowest terms:

$$\frac{3(p-q)}{p} \cdot \frac{q}{2(p-q)}, \frac{x^2+3x}{x^2-3x-4} \cdot \frac{x^2-5x+4}{x^2+2x-3}$$

3. Divide. Write the answer in lowest terms:

$$\frac{9p^2}{3p+4} \div \frac{6p^3}{3p+4}, \frac{x^2-4}{x^2+x-6} \div \frac{x^2+5x+6}{-2x}, \frac{m^2-4}{m^2-1} \div \frac{2m^2+4m}{1-m}$$

4. Add. Write the answer in lowest terms:

$$\frac{2x}{x^2-1} + \frac{-1}{x+1}, \frac{2x}{x^2+5x+6} + \frac{x+1}{x^2+2x-3}$$

5. Subtract. Write the answer in lowest terms:

$$\frac{2m}{m-n} - \frac{5m+n}{2m-2n}, \frac{5}{x^2-9} - \frac{x+2}{x^2+4x+3}$$

6. Simplify the complex fractions. Write the answer in lowest terms:

$$\frac{\frac{4a^4b^3}{3a}}{\frac{2ab^4}{b^2}}, \frac{\frac{2}{p^2} - \frac{3}{5p}}{\frac{4}{p} + \frac{1}{4p}}$$

Each of the above can be turned into a rational equation by setting the expression equal to 1 or some other value or expression and then solving.

Conceptual Understanding

1. Why can't the denominator of a rational expression equal 0?
2. Explain how the LCD is used in a different way when adding and subtracting rational expressions compared to solving equations with rational expressions.
3. Suppose that $(2x-5)^2$ is the LCD for two fractions. Is $(5-2x)^2$ also acceptable as an LCD? Why or why not?

Problem Solving/Application

1. The Tickfaw River has a current of 3 mph. A motorboat takes as long to go 12 miles downstream as to go 8 miles upstream. What is the speed of the boat in still water?
2. In a certain fraction, the denominator is 6 more than the numerator. If 3 is added to both the numerator and the denominator, the resulting fraction is equivalent to $\frac{5}{7}$.
What is the original fraction?

Standard #14

Standard Set 14.0

Students solve a quadratic equation by factoring or completing the square.

Deconstructed Standard

1. Students solve a quadratic equation by factoring.
2. Students solve a quadratic equation by completing the square.

Prior Knowledge Necessary

Students should know:

- how to solve linear equations
- how to verify that a solution to an equation satisfies the original equation
- how to simplify expressions prior to solving an equation
- how to multiply binomials
- how to identify an equation as a quadratic equation
- how to write a quadratic equation in standard form
- how to factor the greatest common factor (GCF) from a polynomial equation
- how to factor a polynomial using grouping
- how to factor quadratic expressions into the product of two binomials
- how to identify a perfect square trinomial
- the Multiplication Property of Zero
- how to evaluate the principle square root of a perfect square
- how to simplify the square root of a non-perfect square

New Knowledge

Students will need to learn how to:

- apply the Principle of Zero Products in obtaining the solutions to a quadratic equation
- identify quadratic equations that have no real solutions
- use the Square Root Property to solve equations involving a binomial squared set equal to a constant (e.g., $(x + 5)^2 = 9$)
- manipulate a quadratic equation to produce a perfect square trinomial on one side of the equation

Categorization of Educational Outcomes

Competence Level: Application

1. Students will use methods of factoring polynomials to solve quadratic equations.
2. Students will use the method of completing the square of a quadratic polynomial to solve a quadratic equation.
3. Students will solve quadratic equations.
4. Students will demonstrate the ability to use the technique of completing the square.
5. Students will show that they know the correct interpretation of the roots or zeroes of a polynomial equation.
6. Students will demonstrate the ability to identify the correct technique to solve a quadratic equation.

Necessary New Physical Skills

1. Students should be capable of using a graphing calculator to identify/verify the solutions(s) of a quadratic equation.

Assessable Result of the Standard

1. Students will generate solutions to a quadratic equation.

Standard #14 Model Assessment Items

Computational and Procedural Skills

1. Solve by factoring:
 - a. $x^2 + 5x + 6 = 0$
 - b. $4p^2 + 40 = 26p$
 - c. $16m^2 - 25 = 0$
2. Solve by completing the square:
 - a. $x^2 + 6x + 7 = 0$
 - b. $4z^2 + 24z - 13 = 0$
 - c. $(x + 3)(x - 1) = 2$

Conceptual Understanding

1. Factoring:
 - a. In order to solve a quadratic equation by factoring, in what form must the equation be?
 - b. If the product of two binomial expressions (e.g., $(2x - 1)(x + 3)$) is set equal to zero, what does the Principle of Zero Products (or Zero-Factor Property) say about the two binomials?
2. Completing the square:
 - a. In order to solve $4z^2 + 24z - 13 = 0$, what is the first step in completing the square?
 - b. In using the method of completing the square to solve $2x^2 - 10x = -8$, a student began by adding the square of half the coefficient of x (that is $\left[\frac{1}{2}(-10)\right]^2 = 25$) to both sides of the equation. He then encountered difficulty in his later steps. What was his error? Explain the steps needed to solve the problem, and give the solution set.

Problem Solving/Application

1. Factoring:
 - a. The O'Connors want to plant a flower bed in a triangular area in a corner of their garden. One leg of the right-triangular flower bed will be 2 m shorter than the other leg, and they want it to have an area of 24 m^2 . Find the lengths of the legs.

- b. The product of two consecutive integers is 11 more than their sum. Find the integers.
2. Completing the square:
- a. If an object is propelled upward on the surface of Mars from ground level with an initial velocity of 104 ft per second, its height s (in feet) in seconds, t is given by the formula $s = -13t^2 + 104t$. How long will it take for the object to be at a height of 195 ft? How long will it take the object to return to the surface (hint: $s = 0$)?
- b. A farmer has a rectangular cattle pen with a perimeter of 350 ft and an area of 7500 ft^2 . What are the dimensions of the pen?

Standard #15

Standard Set 15.0

Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.

Deconstructed Standard

1. Students apply algebraic techniques to solve rate problems.
2. Students apply algebraic techniques to solve work problems.
3. Students apply algebraic techniques to solve percent mixture problems.

Prior Knowledge Necessary

Students should know how to:

- solve linear equations
- solve quadratic equations
- solve rational equations
- translate word problems into algebraic equations

New Knowledge

Students will need to learn how to:

- set up and solve a rate problem using appropriate units
- set up and solve a work problem using appropriate units
- set up and solve a percent mixture problem using appropriate units

Categorization of Educational Outcomes

Competence Level: Application

1. Students will be able to write the appropriate equation and solve work problems related to rate with appropriate units.
2. Students will be able to write the appropriate equation and solve work problems related to work with appropriate units.
3. Students will be able to write the appropriate equation and solve work problems related to percent mixture with appropriate units.

Competence Level: Analysis

1. Students will be able to organize information given in a rate problem.
2. Students will be able to organize information given in a work problem.
3. Students will be able to organize information given in a percent mixture problem.

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will find solutions to rate, work, and percent-mixture problems with appropriate units.

Standard #15 Model Assessment Items

Problem Solving/Application

1. Setting up and solving a rate problem:
 - a. A passenger train leaves Houston at a speed of 60 km/hr. Two hours later a second passenger train leaves Houston going in the same direction as the first train on parallel tracks at 90 km/hr. How many hours later will the two train meet at the same spot?
2. Setting up and solving a work problem:
 - a. Julie can wax her car in 2 hours. When she works together with Martha, they can wax the car in 45 minutes. How long would it take Martha working by herself, to wax the same car?
3. Setting up and solving a percent mixture problem:
 - a. 5% potassium is mixed with 15% potassium to make a fertilizer. How much potassium of each type should be mixed to make 30 gallons of mixture that is 11% potassium?

Standard #16

Standard Set 16.0

Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.

Deconstructed Standard

1. Students understand the definition of a relation.
2. Students understand the concept of a function.
3. Students determine whether or not a given relation is a function.
4. Students give pertinent information (the domain and range) about a relation.
5. Students give pertinent information (the domain and range) about a function.

Prior Knowledge Necessary

Students should know:

- how to solve equations
- how to verify that a solution to an equation satisfies the original equation
- the definition of an ordered pair
- how to graph sets of ordered pairs
- how to graph linear equations

New Knowledge

Students will need to learn:

- how to identify an ordered pair or set of ordered pairs as a relation
- how to identify the components of a relation
- the definition of “domain of a relation”
- the definition of “range of a relation”
- how to identify the domain of a relation from the sets of ordered pairs
- how to identify the range of a relation from the sets of ordered pairs
- the definition of a function
- how to identify if a relation is a function
- how to identify if an equation is a function
- the definition of “domain of a function”
- the definition of “range of a function”
- how to identify the domain of a function
- how to identify the range of a function

Categorization of Educational Outcomes

Competence Level: Knowledge

1. Students will identify the components of a relation.
2. Students will identify the domain of a relation.
3. Students will identify the range of a relation.
4. Students will identify if a relation is a function.
5. Students will examine an equation and identify if it is a function.

Competence Level: Comprehension

1. Students will differentiate between sets of ordered pairs as those that are a function and those that are just a relation.
2. Students will differentiate between equations that are functions and those that are not.

Competence Level: Application

1. Students will show that a set of ordered pairs (a relation) satisfies or does not satisfy the definition of a function.
2. Students will show that an equation satisfies or does not satisfy the definition of a function.

Necessary New Physical Skills

1. Students should be capable of graphing a linear equation.

Assessable Result of the Standard

1. Students will identify if a relation is a function.
2. Students will identify if an equation is a function.
3. Students will identify the domain and range of a given function.

Standard #16 Model Assessment Items

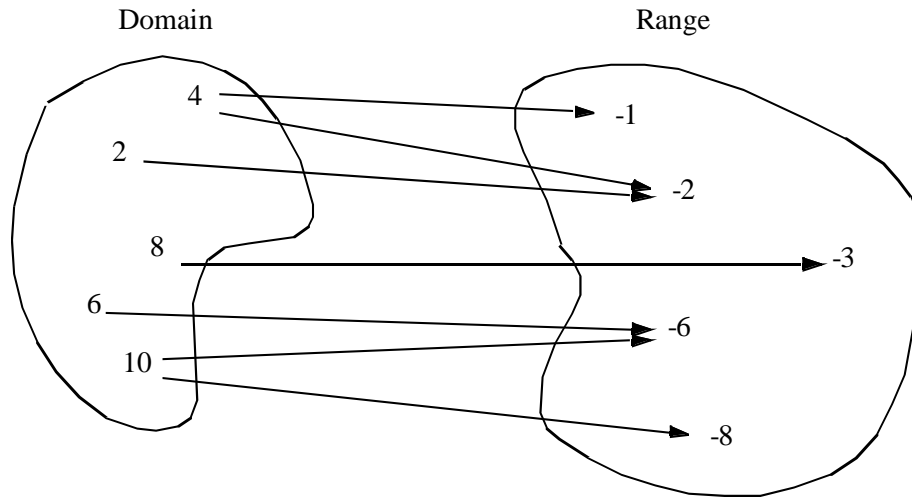
Conceptual Understanding

1. Given the following sets of ordered pairs, give the domain and range of the relation; determine whether the relation *is* or *is not* a function:

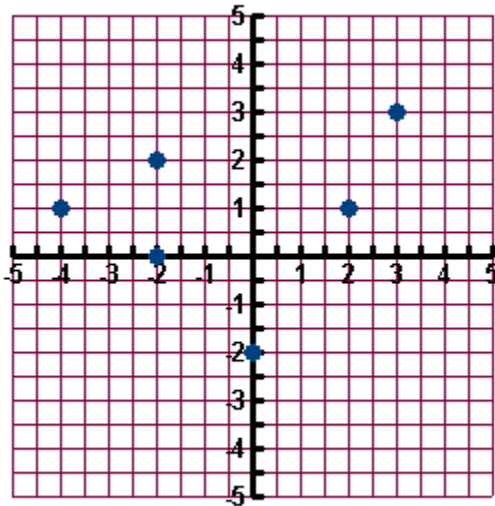
- a. $\{(-4,3), (-2,1), (0,5), (-2,-8)\}$
- b. $\{(3,7), (1,4), (0,-2), (-1,-1), (-2,5)\}$

2. Given the following figures, give the domain and range of the relation; determine whether the relation *is* or *is not* a function:

a.



b.



3. Define *relation* and *function*. Compare the two definitions. How are they alike? How are they different?

Problem Solving/Applications

1. Decide whether each equation defines y as a function of x . (Remember that to be a function, every value of x must give one and only one value of y .)
 - a. $y = 5x + 3$
 - b. $x = y^2$
2. Find the domain and range for each function.
 - a. $y = 3x - 2$
 - b. $y = x^2 - 3$

Standard #17

Standard Set 17.0

Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.

Deconstructed Standard

1. Students determine the domain of independent variables defined by a graph.
2. Students determine the domain of independent variables defined by a set of ordered pairs.
3. Students determine the domain of independent variables defined by a symbolic expression.
4. Students determine the range of dependent variables defined by a graph.
5. Students determine the range of dependent variables defined by a set of ordered pairs.
6. Students determine the range of dependent variables defined by a symbolic expression.

Prior Knowledge Necessary

Students should know how to:

- identify ordered pairs of data from a graph
- graph points on a coordinate plane
- write ordered pairs correctly
- use a variety of methods such as symbols, charts, graphs, tables, and diagrams to explain mathematical reasoning

New Knowledge

Students will need to learn how to:

- identify the domain in each symbolic expression, table, or graph
- identify the range in each symbolic expression, table, or graph

Categorization of Educational Outcomes

Competence Level: Knowledge

1. Students will identify the domain of independent variables defined by a graph, a set of ordered pairs, or a symbolic expression.
2. Students will identify the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.

Competence Level: Comprehension

1. Students will be able to describe the difference between the domain of independent variables and the range of dependent variables.

Competence Level: Application

1. Students will be able to examine data from a graph, a set of ordered pairs, or a symbolic expression and classify it into the domain of independent variables or range of dependent variables.

Competence Level: Analysis

1. Students will be able to analyze data from a variety of sources and categorize the data into the domain of independent variables or range of dependent variables.

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Once given the domain of independent variables and range of dependent variables, the student will plot the ordered pairs.

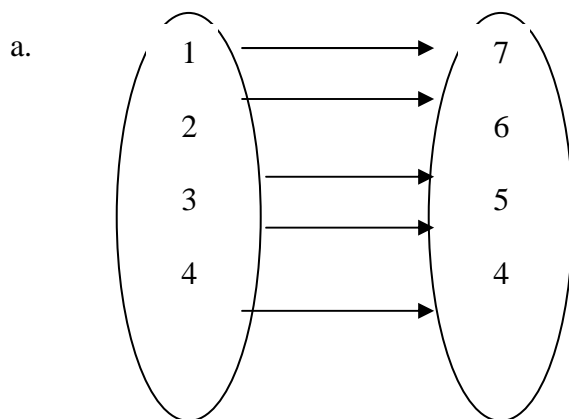
Standard #17 Model Assessment Items

Computational and Procedural Skills

1. Complete each ordered pair so that it is a solution to $3x + y = 10$. Then identify the domain and range of each ordered pair:
 - a. (1, ?)
 - b. (2, ?)
 - c. (?, 4)
 - d. (3, ?)
 - e. (0, ?)

Conceptual Understanding

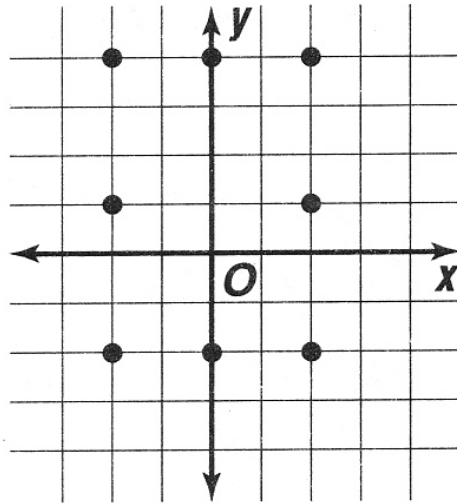
1. State the domain and range of each relation:
 - a. { (2,4), (2,5), (4,6), (7,2), (5,10), (8,4), (3,6) }
2. Express the relation in each mapping, table, or graph as a set of ordered pairs and then state the domain and range of each:



b.

x	y
3	4
5	6
7	8
7	6
5	4
3	3

c.



Problem Solving/Application

- The table below shows the amount that a company charges for a bike rental. Identify the domain and range. Write a set of ordered pairs for the function. Write an equation for the function.

a.

Time (hrs)	1	2	3	4	5	6
Cost (\$)	20	24	28	32	36	40

- The table below shows the per-minute rate for a cell phone. Identify the domain and range. Write a set of ordered pairs for the function. Write an equation for this function.

b.

Minutes	1	2	3	4	5	6
Cost (\$)	2.00	2.25	2.50	2.75	3.00	3.25

- The table below shows the distance that a car travels over time. Identify the domain and range. Write a set of ordered pairs for the function. Use the table to write an equation for this function.

c.

Time (hrs)	1	2	3	4	5	6
Distance (miles)	50	100	150	200	250	300

Standard #18

Standard Set 18.0

Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.

Deconstructed Standard

1. Given a graph, students can determine if the relation graphed is also a function.
2. Students can justify why the graph of a relation is or is not a function.
3. Students can determine if the relation defined by a set of ordered pairs is also a function.
4. Students can justify why the set of ordered pairs is or is not a function.
5. Given an algebraic equation, students can determine whether or not it represents a function.
6. Students can justify why the algebraic equation is or is not a function.

Prior Knowledge Necessary

Students should know:

- the definition of a relation
- the definition of a function
- the definition of *range*
- the definition of *domain*
- how to determine the domain of the independent variable from a graph, a set of ordered pairs, or a symbolic expression (standard 17)
- how to determine the range of the dependent variable from a graph, a set of ordered pairs, or a symbolic expression (standard 17)
- how to determine whether a given relation defines a function (standard 16)
- how to state exclusions on the variable when given a rational expression
- how to state exclusions on the variable when given a radical expression
- how to give the ordered pairs plotted in an x - y grid

New Knowledge

Students will need to learn:

- how to use the vertical line test to determine if a given graph represents a function
- how to justify their conclusion as to whether a given graph, set of ordered pairs, or symbolic expression is a function

Categorization of Educational Outcomes

Competence Level: Knowledge

1. Students will identify whether or not a given graph, set of ordered pairs or symbolic expression is a function.
2. Students will use the definition of a function to justify their response.

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will state whether or not a given graph, set of ordered pairs, or algebraic equation is a function and justify their answer.

Standard #18 Model Assessment Items

Computational and Procedural Skills

1. Determine whether or not the below relations are functions. Justify your answer.

a. $\{(4, 5), (-3, 6), (5, 6), (-2, 4)\}$

b. $\{(7, -3), (-3, 6), (7, -3), (-6, 5)\}$

c. $y = 3x - 1$

d. $f(x) = x^2 - 5$

e. $x = y^2 - 3$

f. $y = \sqrt{x - 5}$

g. $x^2 + y^2 = 16$

Conceptual Understanding

1. If you are looking at a graph, how do you determine whether or not it is the graph of a function?
2. Write a set consisting of three ordered pairs that is a relation, but not a function. Explain why the set of ordered pairs you wrote is not a function.
3. Draw a graph that is both a relation and a function. Write a sentence that states why the graph you drew represents both a relation and a function.

Standard #19

Standard Set 19.0

Students know the quadratic formula and are familiar with its proof by completing the square.

Deconstructed Standard

1. Students know the quadratic formula.
2. Students know how to solve a quadratic equation by using the quadratic formula.
3. Students are familiar with the proof, or derivation, of the quadratic formula by completing the square.

Prior Knowledge Necessary

Students should know:

- how to solve a quadratic equation by factoring
- the Property of Zero as it pertains to solving quadratic equations by the factoring method
- the standard form for a quadratic equation as it pertains to the use of the quadratic formula: $ax^2 + bx + c = 0$
- how to solve linear equations
- how to evaluate the square root of a number that is a perfect square
- how to simplify the square root of a number that is not a perfect square
- how to use a calculator to approximate and round the square root of a non-perfect square
- how to solve a quadratic equation by completing the square

New Knowledge

Students will need to:

- memorize the quadratic formula
- learn when it is appropriate to use the quadratic formula vs. other techniques to solve a quadratic equation
- learn how to solve a quadratic equation by using the quadratic formula
- learn how the quadratic formula is derived by the method of completing the square

Categorization of Educational Outcomes

Competence Level: Application

1. Students will be able to recall and apply the quadratic formula to solve quadratic equations.

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will produce solutions to quadratic equations, in both approximate and exact simplified forms, by using the quadratic formula.

Standard #19 Model Assessment Items

Computational and Procedural Skills

1. State the quadratic formula.
2. Solve the following quadratic equation by using the quadratic formula:
 $2x^2 + 7x - 15 = 0$
3. Solve the following quadratic equation and approximate your answers to the nearest hundredth: $3x^2 = -4x + 1$
4. Solve the following quadratic equation and record your answer in exact simplified form: $x^2 + (x + 2)^2 = 7$

Conceptual Understanding

1. In what situations would it be advantageous to use the quadratic formula to solve a quadratic equation?
2. List three techniques, other than the quadratic formula, for solving quadratic equations. Create an example to illustrate each technique.
3. Without actually proving/deriving the quadratic formula, explain in your own words how the quadratic formula can be derived by using the technique known as Completing the Square.

Problem Solving/Application

1. The length of a rectangle is 3 meters greater than twice the width. The area of this rectangle is 125 square meters. Find the length and width of this rectangle (round your answer to the nearest hundredth if necessary).
2. A right triangle is such that one leg is 3 feet longer than the other leg. The length of the hypotenuse is 17 feet. Find the lengths of the legs of this right triangle (round your answers to the nearest hundredth if necessary).

Standard #20

Standard Set 20.0

Students use the quadratic formula to find the roots of a second-degree polynomial and to solve quadratic equations.

Deconstructed Standard

1. Students use the quadratic formula.
2. Students find the roots of a second-degree polynomial.
3. Students solve quadratic equations.

Prior Knowledge Necessary

Students should know how to:

- apply algebraic order of operations and the commutative, associative, and distributive properties to evaluate expressions
- solve problems manually by using the correct order of operations or on a scientific calculator
- identify the degree of a polynomial
- differentiate between linear and quadratic functions
- evaluate formulas for given formula values
- simplify radical expressions
- simplify square roots
- evaluate for given values of x
- evaluate expressions for given values of x

New Knowledge

Students will need to learn how to:

- substitute appropriate a , b , and c values into the quadratic equation
- simplify the resulting expressions
- recognize that the roots of a second degree polynomial are the solutions to the quadratic equation

Categorization of Educational Outcomes

Competence Level: Knowledge

1. Students will be able to identify the vertex of a quadratic function.
2. Students will be able to identify the maximum or minimum value of the quadratic function.
3. Students will be able to identify the axis of symmetry of a quadratic function.
4. Students will be able to identify the x -intercepts of a quadratic equation.

Competence Level: Comprehension

1. Students will be able to describe how to locate the vertex of a quadratic function and identify the maximum or minimum value.
2. Students will be able to describe how to locate the axis of symmetry of a quadratic function.

Competence Level: Application

1. Students will be able to calculate the vertex of a quadratic function.
2. Students will be able to calculate the axis of symmetry of a quadratic function.
3. Students will be able to calculate the x -intercepts of a quadratic equation.

Competence Level: Analysis

1. Students will be able to explain the process of graphing a quadratic formula.

Competence Level: Synthesis

1. Students will predict the minimum or maximum values of the quadratic function before graphing the function.

Competence Level: Evaluation

1. Students will be able to verify their solution by graphing the quadratic equation and comparing their answer to the x -intercepts.

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will produce the roots of a quadratic equation using the quadratic formula.

Standard #20 Model Assessment Items

Computational and Procedural Skills

1. Solve using the quadratic formula:

a. $x^2 + 7x + 12 = 0$

b. $2x^2 + 3x + 1 = 0$

c. $8x^2 + 18x - 5 = 0$

d. $-x^2 - x + 12 = 0$

Problem Solving/Application

1. The area of a rectangle is 44 inches. The perimeter of the rectangle is 30 inches. Find the length and width.
2. A company produces guitars. The function for the profit of the company is:
 $P(x) = -.3x^2 + 75x - 2000$. Find the break-even points (the selling prices for which the profit is 0).
3. The length of a rectangle is 6 feet longer than its width. If the area is 50 square feet, find the length and width of the rectangle.

Standard #21

Standard Set 21.0

Students graph quadratic functions and know that their roots are the x -intercepts.

Deconstructed Standard

1. Students graph quadratic functions.
2. Students know that the roots of a quadratic equation are the x -intercepts.

Prior Knowledge Necessary

Students should know how to:

- differentiate between linear and quadratic functions
- graph linear functions
- plot points on a coordinate system

New Knowledge

Students will need to learn how to:

- calculate the axis of symmetry of a quadratic function using the formula $x = -b/2a$
- calculate the vertex of a quadratic function by plugging the x value into the quadratic equation and solving for y
- graph the axis of symmetry using the equation $x = -b/2a$
- identify whether the parabola opens upward or downward

Categorization of Educational Outcomes

Competence Level: Knowledge

1. Students will be able to identify the vertex of a quadratic function.
2. Students will be able to identify the maximum or minimum value of the quadratic function.
3. Students will be able to identify the axis of symmetry of a quadratic function.
4. Students will be able to identify the x -intercepts of a quadratic equation.

Competence Level: Comprehension

1. Students will be able to describe how to locate the vertex of a quadratic function and identify the maximum or minimum value.
2. Students will be able to describe how to locate the axis of symmetry of a quadratic function.

Competence Level: Application

1. Students will be able to calculate the vertex of a quadratic function.
2. Students will be able to calculate the axis of symmetry of a quadratic function.
3. Students will be able to calculate the x -intercepts of a quadratic equation.

Competence Level: Analysis

1. Students will be able to explain the process of graphing a quadratic formula.

Competence Level: Synthesis

1. Students will predict the minimum or maximum values of the quadratic function before graphing the function.

Competence Level: Evaluation

1. Students will be able to verify their solution by graphing the quadratic equation and comparing their answer to the x -intercepts.

Necessary New Physical Skills

1. Students use a ruler to draw the x -axis and y -axis.

Assessable Result of the Standard

1. Students will graph a quadratic function with the roots, axis of symmetry, and vertex.

Standard #21 Model Assessment Items

Computational and Procedural Skills

1. Write the equation of the axis of symmetry:
 - a. $y = 4x^2$
 - b. $y = -x^2 + 4x - 1$
 - c. $y = x^2 - 3x - 10$
 - d. $y = 2x^2 + 16$
2. Find the coordinates of the vertex and state whether the vertex is a maximum or a minimum
 - a. $y = x^2 + 2$
 - b. $y = -x^2 - 6x + 9$
 - c. $y = -x^2 + 5x + 6$
 - d. $y = x^2 - 14x + 13$

Conceptual Understanding

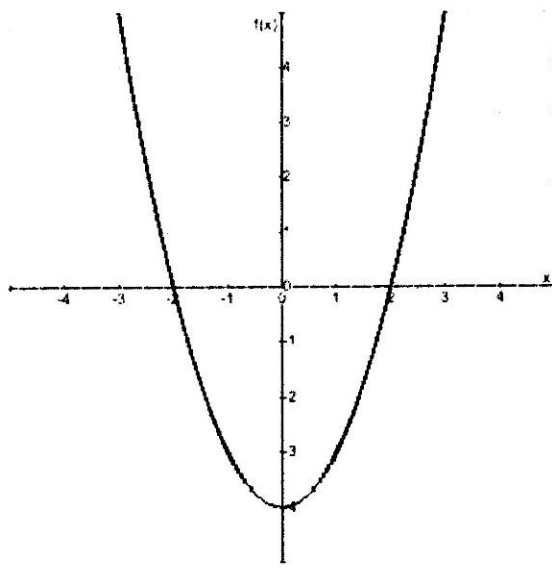
1. Identify the roots from the following graphs:



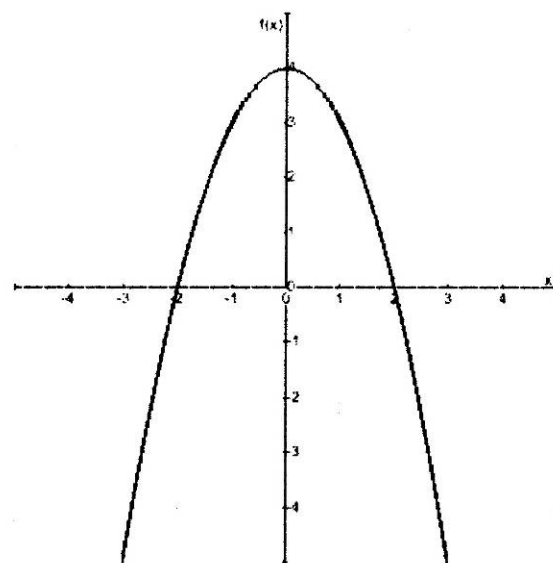
Assessment
Conceptual Understanding

Identify the roots from the following graphs

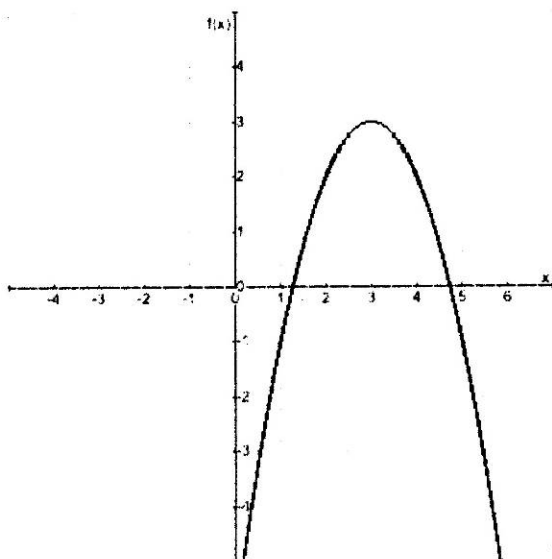
1.



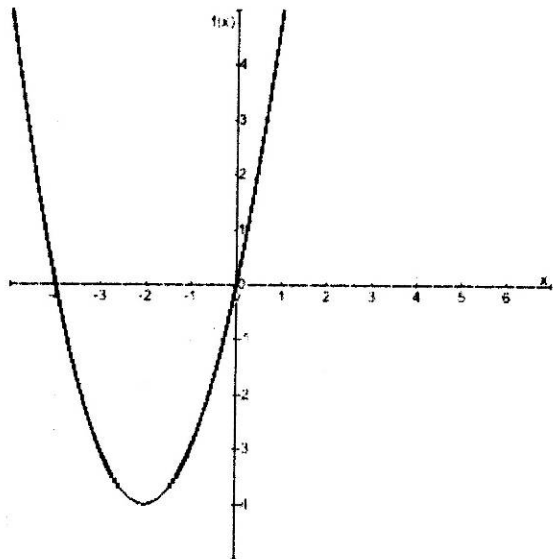
2.



3.



4.



Problem Solving/Application

1. An object is thrown straight upward with an initial velocity of 30 feet per second. Its height in feet, y , after t seconds, is represented by: $y = -16t^2 + 32t$. In how many seconds does the object reach its maximum height?
2. George punts a football. He can kick the ball with an upward velocity of 80 feet per second and his foot meets the ball 2 feet off the ground.
 - a. Write a quadratic equation to describe the height of the football at any given time. (Use 32 ft/sec^2 as the acceleration of gravity.)
 - b. How high is the ball after 1 second?
 - c. When does the ball hit the ground?

Exemplar Teaching Practices

To celebrate its championship Junior Varsity soccer team, Jefferson High is planning a fireworks display. The fireworks are to be created using rockets that will be launched from the top of a tower near the school. The top of the tower is 160 feet off the ground. The launching mechanism will shoot the rockets so that they are initially rising at 92 feet per second. The soccer team wants the fireworks from each rocket to explode when the rocket is at the top of its trajectory. They need to know how long it will take for the rocket to reach the top so they can set the timing mechanism. Also, in order to tell the people where they can stand to see the display, they need to know how high the rockets will get. The rockets will be aimed toward an empty field and shot at an angle of 35 degrees above the horizontal. The team wants to know how far the rockets will land from the base of the tower so they can fence off the area in advance.

One student, Hilda, says that there is a function $h(t)$ that will give the rocket's height off the ground in terms of the time t elapsed since the launch: $h(t) = 160 + 92t - 16t^2$. Hilda also says that the team can find the horizontal distance the rocket travels with the formula: $d(t) = 92 \cos 35^\circ t$. (Again, t is the number of seconds since the rocket was launched, and $d(t)$ is the distance measured in feet.)

Help the soccer team find the answers to their questions by completing the following:

1. Draw a sketch of the situation.
2. Write a clear statement of the questions the soccer team wants answered.
3. Describe how you might use Hilda's formulas to help answer the questions stated in Question 2.
4. Using whatever methods you choose, try to get some answers to the questions you stated in Question 2.

The above problem taken from the textbook: CPM 1:Fireworks Unit

Standard #22

Standard Set 22.0

Students use the quadratic formula or factoring techniques or both to determine whether the graph of a quadratic function will intersect the x -axis in zero, one, or two points.

Deconstructed Standard

1. Students use the quadratic formula to solve quadratic equations.
2. Students use factoring techniques to solve quadratic equations.
3. Students determine the number of points that the graph of a quadratic function intersects the x -axis.

Prior Knowledge Necessary

Students should know:

- how to identify an equation as a quadratic equation
- how to write a quadratic equation in standard form
- how to factor the greatest common factor (GCF) from a polynomial equation
- how to factor a polynomial using grouping
- how to factor quadratic expressions into the product of two binomials
- how to identify a perfect square trinomial
- the Multiplication Property of Zero
- the relationship between the solutions of a quadratic equation and the roots or zeroes of a quadratic function
- how to graph a quadratic equation and the meaning of roots/zeroes in relationship to the graph
- how to apply the Principle of Zero Products in obtaining the solutions to a quadratic equation
- how to find the solutions to a quadratic equation using the quadratic formula
- the definition of the discriminant and how to use it

New Knowledge

Students will need to learn:

- how to identify quadratic equations that have no real solutions
- how to use the discriminant to identify whether the quadratic equation has zero, one, or two real solutions
- how to use factoring techniques to determine whether the quadratic equation has zero, one, or two real solutions
- how to use the results from the statements above to identify on the graph of the quadratic equation the number of times that the graph of the equation crosses the x -axis

Categorization of Educational Outcomes

Competence Level: Application

1. Students will use methods of factoring polynomials to solve a quadratic equation.
2. Students will use the quadratic formula to solve a quadratic equation.

3. Students will use the method of completing the square of a quadratic polynomial to solve a quadratic equation.
4. Students will demonstrate the ability to identify the correct technique to solve a quadratic equation.
5. Students will solve quadratic equations.
6. Students will demonstrate the ability to use the quadratic formula to solve a quadratic equation.
7. Students will show that they know the correct interpretation of the *roots* or *zeroes* of a quadratic equation.
8. Students will demonstrate the relationship between the discriminant and the number of roots/zeroes of a quadratic equation.
9. Students will demonstrate knowledge of the relationship between the roots/zeroes of a quadratic equation and how many times the graph of the equation intersects the x -axis.

Necessary New Physical Skills

1. Students will graph the quadratic equation, either by hand or by using a graphing calculator, to identify/verify the solutions(s) of a quadratic equation and how many times the equation intersects the x -axis.

Assessable Result of the Standard

1. Students will identify the number of solutions to a quadratic equation and how many times the graph of the equation intersects the x -axis.

Standard #22 Model Assessment Items

Computational and Procedural Skills

1. Solve by factoring and determine the number of times the graph of the equation intersects the x -axis:
 - a. $x^2 + 5x + 6 = 0$
 - b. $4p^2 + 40 = 26p$
 - c. $16m^2 - 25 = 0$
 - d. $x^2 + 6x + 9 = 0$
2. Solve using the quadratic equation and determine the number of times the graph of the equation intersects the x -axis:
 - a. $2x^2 - 7x - 9 = 0$
 - b. $3x^2 + 4x - 8 = 0$
 - c. $\frac{1}{2}x^2 + \frac{1}{6}x = 1$

Conceptual Understanding

1. Factoring:
 - a. In solving a quadratic equation that is a perfect square trinomial, how many times does the graph of the equation intersect the x -axis?
 - b. In factoring a quadratic equation and there are no solutions, what does that mean for the graph of the quadratic equation?
1. Quadratic Formula:
 - a. What is the relationship between the discriminant and the number of times the graph of the quadratic equation intersects the x -axis?
 - b. If the quadratic formula results in only one solution, what does the graph look like in relationship to the x -axis?

Problem Solving/Application

1. Factoring:
 - a. The length of a rug is 6 ft more than the width. The area is 40 ft^2 . Find the length and width of the rug.
 - b. The volume of a rectangular box is 120 m^3 . The width of the box is 4 m, and the height is 1 m less than the length. Find the length and height of the box.

1. Quadratic Formula:
 - a. A frog is sitting on a stump 3 ft above the ground. He hops off the stump and lands on the ground 4 ft away. During his leap, his height h is given by the equation $h = -0.5x^2 + 1.25x + 3$, where x is the distance in feet from the base of the stump, and h is in feet. How far was the frog from the base of the stump when he was 1.25 ft above the ground?
 - b. Solve the formula $S = 2\pi rh + \pi r^2 h$ for r by writing it in the form $ar^2 + br + c = 0$, and then using the quadratic formula.

Standard #23

Standard Set 23.0

Students apply the quadratic equations to physical problems, such as the motion of an object under the force of gravity.

Deconstructed Standard

1. Students use the quadratic formula to solve quadratic equations describing physical problems.
2. Students use factoring techniques to solve quadratic equations describing physical problems.

Prior Knowledge Necessary

Students should know:

- how to translate a word problem describing a physical situation into a quadratic equation, identifying the unknown quantity and assigning a variable to it
- how to write a quadratic equation in standard form
- the Multiplication Property of Zero
- how to apply the Principle of Zero Products in obtaining the solutions to a quadratic equation
- how to find the solutions to a quadratic equation using the quadratic formula
- how to solve quadratic equations using the different methods, such as the different factoring techniques and completing the square

New Knowledge

Students will need to learn how to:

- use the discriminant to identify whether the quadratic equation has zero, one, or two real solutions
- use factoring techniques to determine whether the quadratic equation has zero, one, or two real solutions
- use the results from the two statements above and knowledge of the problem to obtain solutions to physical problems (i.e., discarding unrealistic solutions)

Categorization of Educational Outcomes

Competence Level: Application

1. Students will use methods of factoring polynomials to solve a quadratic equation.
2. Students will use the quadratic formula to solve a quadratic equation.
3. Students will use the method of completing the square of a quadratic polynomial to solve a quadratic equation.
4. Students will demonstrate the ability to identify the correct technique to solve a quadratic equation.
5. Students will solve quadratic equations.
6. Students will demonstrate the ability to use the quadratic formula to solve physical problems involving a quadratic equation.

7. Students will identify correct solutions by rejecting physically impossible solutions to quadratic equations describing physical problems.

Necessary New Physical Skills

None

Assessable Result of the Standard

1. Students will identify the solutions to a quadratic equation describing physical problems.

Standard #23 Model Assessment Items

Conceptual Understanding

1. The solutions to a quadratic equation involving the length of an object include a negative solution. Is this physically possible? What does a negative solution mean in this context?

Problem Solving/Application

1. The length of a rug is 6 ft more than the width. The area is 40 ft^2 . Find the length and width of the rug.
2. The volume of a rectangular box is 120 m^3 . The width of the box is 4 m, and the height is 1 m less than the length. Find the length and height of the box.
3. Galileo's formula for freely falling objects is given as $d = 16t^2$. The distance d in feet that an object falls depends on the time elapsed (t) in seconds.
 - a. Use Galileo's formula and complete the following table (*Hint*: substitute each given value into the formula and solve for the unknown value):

t in seconds	0	1	2	3		
d in feet	0	16			256	576

- i. When $t = 0$, $d = 0$. Explain this in the context of the problem.
 - ii. When you substituted 256 for d and solved for t , you should have found two solutions, 4 and -4. Why doesn't -4 make sense as an answer?
- b. Tram works due north of home. Her husband Alan works due east. They leave for work at the same time. By the time Tram is 5 miles from home, the distance between them is 1 mile more than Alan's distance from home. How far from home is Alan? (*Hint*: draw a picture.)

(These problems were taken from *Beginning Algebra*, 9th Edition by Lial, Hornsby, and McGinnis; published by Pearson Addison-Wesley, 2004. Chapter 6, Section 6.)

Appendix #1

CALIFORNIA CONTENT STANDARDS: ALGEBRA I	
Standard Set 1.0: Students identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers, including closure properties, for the four basic arithmetic operations where applicable.	
1.1: Students use properties of numbers to demonstrate whether assertions are true or false.	
2.0*: Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.	
3.0: Students solve equations and inequalities involving absolute values.	
4.0*: Students simplify expressions prior to solving linear equations and inequalities in one variable, such as $3(2x-5) + 4(x-2) = 12$.	
5.0*: Students solve multi-step problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.	
6.0*: Students graph a linear equation and compute the x- and y- intercepts (e.g., graph $2x + 6y = 4$). They are also able to sketch the region defined by linear inequality (e.g., they sketch the region defined by $2x + 6y < 4$).	
7.0*: Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations using the point-slope formula.	
8.0: Students understand the concepts of parallel lines and perpendicular lines and how those slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.	
9.0*: Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.	
10.0*: Students add, subtract, multiply, and divide monomials and polynomials. Students solve multi-step problems, including word problems, by using these techniques.	
11.0: Students apply basic factoring techniques to second- and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.	
12.0*: Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.	
13.0*: Students add, subtract, multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems by using these techniques.	
14.0*: Students solve a quadratic equation by factoring or completing the square.	

15.0*: Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.
16.0: Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.
17.0: Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.
18.0: Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.
19.0*: Students know the quadratic formula and are familiar with its proof by completing the square.
20.0*: Students use the quadratic formula to find the roots of a second-degree polynomial and to solve quadratic equations.
21.0*: Students graph quadratic functions and know that their roots are the x-intercepts.
22.0: Students use the quadratic formula or factoring techniques or both to determine whether the graph of a quadratic function will intersect the x-axis in zero, one, or two points.
23.0*: Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity.
<p>24.0: Students use and know simple aspects of a logical argument.</p> <p>24.1: Students explain the difference between inductive and deductive reasoning and identify and provide examples of each.</p> <p>24.2: Students identify the hypothesis and conclusion in logical deduction.</p> <p>24.3: Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.</p>
<p>25.0: Students use properties of the number system to judge the validity of results, to justify each step of a procedure, and to prove or disprove statements.</p> <p>25.1: Students use properties of numbers to construct simple, valid arguments (direct and indirect) for, or formulate counterexamples to, claimed assertions.</p> <p>25.2: Students judge the validity of an argument according to whether the properties of the real number system and the order of operations have been applied correctly at each step.</p> <p>25.3: Given a specific algebraic statement involving linear, quadratic, or absolute value expressions or equations or inequalities, students determine whether the statement is true sometimes, always, or never.</p>

Appendix #2

Developing Learning Targets for Algebra Standards (Instructions given to teachers involved in the project)

Please note the following terms and/or definitions which have been agreed upon for this deconstruction project:

- **Prior Knowledge:** Prior knowledge is defined as acquired knowledge that has been mastered in a previous standard.
- **New Knowledge:** New knowledge is defined as knowledge that students need to acquire and apply to the components in step #2 of this deconstruction process to create the products listed in step #7 of this process.
- **Introduced:** When a standard mentions that a concept or idea has been “introduced,” this does not mean that it has been mastered.
- **Familiar With:** When a standard mentions that students should be “familiar with” certain concepts or ideas, this does not mean that students have actually mastered these ideas or concepts.

Sample:

Deconstruction of Standard #6

Step #1: Underline noun phrases, and box or circle the verbs.

Standard 6.0: Students graph a linear equation and compute the x- and y-intercepts. (e.g., graph $2x + 6y = 4$). They are also able to sketch the region defined by a linear inequality (e.g., they sketch the region defined by $2x + 6y < 4$).

Step #2: Rewrite standard into short components.

1. Students graph linear equations.
2. Students compute x-intercepts.
3. Students compute y-intercepts.
4. Students use intercepts to graph linear equations.
5. Students sketch the region defined by a linear inequality.

Step #3: Identify prior knowledge students should know. (See note above)

1. Students must be able to perform arithmetic computations with rational numbers.
2. Students must be able to graph ordered pairs.
3. Students must be able to compute slope from the graph of a line.
4. Students must be able to compute slope when given two points.
5. Students must be able to recognize slope as a rate of change of y in relation to x.
6. Students must be able to graph a linear equation using a “t-chart”.
7. Students must be able to evaluate a linear equation for a given x or y value.
8. Students must be able to solve one-variable linear inequalities.
9. Students must be able to graph the solution set for a one-variable linear inequality.
10. Students must be able to verify that any element in the solution set of a one-variable inequality satisfies the original inequality.

Step #4: Identify what new knowledge students will need to learn. (See note above)

1. Given the slope/intercept form of a line, $y = mx + b$, students will plot the y-intercept, and then use the slope to find a second point in order to complete the graph of the line.
2. Students will identify the graphical representation of $(a, 0)$ as the x-intercept, and $(0, b)$ as the y-intercept.
3. Students will be able to compute the x-intercept and y-intercept given a linear equation.
4. Students will identify that the linear equation implied by the linear inequality forms a boundary for the solution set and that this boundary may or may not be included in the final graph.
5. Students will interpret the inequality symbol to determine whether or not the boundary is solid or dashed.
6. Students will identify and shade the region of the graph that contains the solutions to the inequality.
7. Students will recognize that linear inequalities have multiple ordered-pair solutions.

Step #5: Identify patterns of reasoning using Bloom's Taxonomy.

Use the *Bloom's Taxonomy* handouts provided to describe the overall competence level expected of students with respect to these topics. Then highlight the "skills demonstrated" using as many of the key words and phrases provided on the handouts. See example below for Standard #6. Box in the key words or phrases taken from *Bloom's Taxonomy*.

Competence Level: Application

1. Students will use methods they have learned to graph lines, solve inequalities, and to locate and/or identify the x- and y-intercepts for a given equation or graph.
2. Students will demonstrate their ability to find and use x- and y-intercepts in the context of graphing.
3. Students will calculate x- and y-intercepts.
4. Students will solve inequalities in two variables.
5. Students will use information they have learned to graph lines, solve inequalities, and find x- and y-intercepts.
6. Students will show that they know the correct interpretation of the boundary line for the solution of an inequality by appropriately making the boundary solid or dashed.

Step #6: Identify required physical skills.

In this section we are looking for physical skills such as: use of a calculator, protractor, ruler, compass, etc.

1. Use of a ruler

Step #7: Identify assessable results of the standard.

1. Students will produce the graph of a line.
2. Students will produce the ordered pairs representing the x- and y-intercepts.
3. Students will produce a bounded and shaded region of the x-y plane representing the solution set of a linear inequality in two variables.

Model Assessment Items

In this section you will write model, or exemplar, assessment items that will serve to demonstrate the level and depth of instruction for these particular topics. For example, you would expect a lower level and depth for a topic in Basic Algebra than you would for the same topic in Intermediate Algebra.

Please be sure to include assessment items that measure abilities in the following three categories:

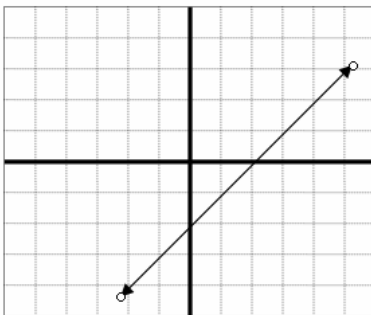
1. Computational and Procedural Skills
2. Conceptual Understanding
3. Problem Solving/Application

Category #1: Computational and Procedural Skills

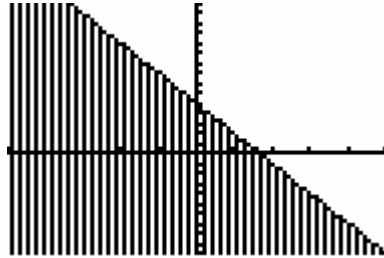
1. Find the x - and y -intercepts for the line defined by the following equation:
 $2x + 3y = 9$
2. Use the x - and y -intercepts to graph the line given by the equation: $2x + 3y = 9$
3. Graph the following lines using the method of your choice. Identify and label the x - and y -intercepts for each graph if they exist:
 - a. $3x - 5y = 10$
 - b. $y = \frac{-2}{3}x + 4$
 - c. $y = 2$
 - d. $x = 3.5$
 - e. $2x + 4y = 3$
 - f. $\frac{1}{2}x - \frac{3}{4}y = 2$
4. Graph the solution set for the following inequalities:
 - a. $2x - 3y < 6$
 - b. $y \geq \frac{-3}{4}x + 2$
 - c. $\frac{1}{2}x - \frac{2}{3}y \leq \frac{5}{6}$

Category #2: Conceptual Understanding

1. Sketch the graph of a line that has no x -intercept.
2. Identify the x - and y - intercepts from the graph of the given line.



3. Can a line have more than one x -intercept? Explain your answer using a diagram.
4. The solution to an inequality has been graphed correctly below. Insert the correct inequality symbol in the inequality below to match the graph of the solution.
(Everything else about the inequality is correct—it just needs the correct symbol).



$$y \boxed{\phantom{<}} 3x + 5$$

Insert correct symbol in box.

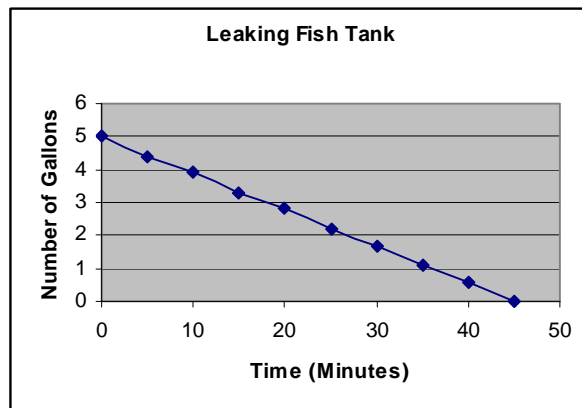
5. When is it advantageous to use the x - and y -intercepts to graph the equation of a line? When would it perhaps be easier or better to use another graphing method? Give an example to illustrate your answers to both of these questions.

Category #3: Problem Solving/Application

1. The graph displayed below is the graph of the following equation: $y = \left(-\frac{1}{9}\right)x + 5$,

where x represents the amount of time that has passed since a 5 gal. fish tank sprung a leak, and y represents the number of gallons of water in the tank after the leak.

- a. What is the significance of the x -intercept in this situation? What information is given to us by this point?
- b. What is the significance of the y -intercept in this situation? What information is given to us by this point?



2. The cost of a trash pickup service is given by the following formula: $y = 1.50x + 11$, where x represents the number of bags of trash the company picks up, and y represents the total cost to the customer for picking up the trash.
 - a. What is the y -intercept for this equation?
 - b. What is the significance of the y -intercept in this situation? What does it tell us about this trash pickup service?
 - c. Draw a sketch of the graph which represents this trash pickup service.

Exemplar Teaching Methods

Please record an example of “Lesson Plans” that demonstrate an excellent method of how the concepts in this standard could be presented to students. Be sure to give examples or illustrate any unique or creative methods that you have used that bring these concepts to life. Use as much detail as needed to communicate your ideas.

Appendix #3

Categorization of Educational Outcomes

Identifies the type of reasoning students will use to learn the skills necessary to master each standard. Teachers were asked to use *Bloom's Taxonomy* to describe the overall competence level expected of students with respect to these topics and highlight the skills demonstrated.

Major Categories in the Taxonomy of Educational Objectives, Bloom 1956*

Categories in the Cognitive Domain: (with Outcome-Illustrating Verbs)

Knowledge—of terminology; specific facts; ways and means of dealing with specifics (conventions, trends, and sequences; classifications and categories; criteria, methodology); universals and abstractions in a field (principles and generalizations, theories and structures)—**The remembering (recalling) of appropriate, previously learned information.**

- defines; describes; enumerates; identifies; labels; lists; matches; names; reads; records; reproduces; selects; states; views.

Comprehension: Grasping (understanding) the meaning of informational materials.

- classifies; cites; converts; describes; discusses; estimates; explains; generalizes; gives examples; makes sense out of; paraphrases; restates (in own words); summarizes; traces; understands.

Application: The use of previously learned information in new and concrete situations to solve problems that have single or best answers.

- acts; administers; articulates; assesses; charts; collects; computes; constructs; contributes; controls; determines; develops; discovers; establishes; extends; implements; includes; informs; instructs; operationalizes; participates; predicts; prepares; preserves; produces; projects; provides; relates; reports; shows; solves; teaches; transfers; uses; utilizes.

Analysis: The breaking down of informational materials into their component parts, examining (and trying to understand the organizational structure of) such information to develop divergent conclusions by identifying motives or causes, making inferences, and/or finding evidence to support generalizations.

- breaks down; correlates; diagrams; differentiates; discriminates; distinguishes; focuses; illustrates; infers; limits; outlines; points out; prioritizes; recognizes; separates; subdivides.

Synthesis: Creatively or divergently applying prior knowledge and skills to produce a new or original whole.

- adapts; anticipates; categorizes; collaborates; combines; communicates; compares; compiles; composes; contrasts; creates; designs; devises; expresses; facilitates; formulates; generates; incorporates; individualizes; initiates; integrates; intervenes;

models; modifies; negotiates; plans; progresses; rearranges; reconstructs; reinforces; reorganizes; revises; structures; substitutes; validates.

Evaluation: Judging the value of material based on personal values/opinions, resulting in an end product, with a given purpose, without real right or wrong answers.

- appraises; compares & contrasts; concludes; criticizes; critiques; decides; defends; interprets; judges; justifies; reframes; supports.

* <http://faculty.washington.edu/krumme/guides/bloom.html>

Appendix #4

Exemplar Practices

Teachers were asked to record an example of “Lesson Plans” that demonstrate an excellent method of how the concepts in this standard could be presented to students. They were asked to be sure to give examples or illustrate any unique or creative methods that they have used that bring these concepts to life, and to use as much detail as needed to communicate their ideas. The following is an example:

Exemplar Teaching Practices for Standard 21.0

To celebrate its championship Junior Varsity soccer team, Jefferson High is planning a fireworks display. The fireworks are to be created using rockets that will be launched from the top of a tower near the school. The top of the tower is 160 feet off the ground. The launching mechanism will shoot the rockets so that they are initially rising at 92 feet per second. The soccer team wants the fireworks from each rocket to explode when the rocket is at the top of its trajectory, they need to know how long it will take for the rocket to reach the top, so they can set the timing mechanism. Also, in order to tell the people where they can stand to see the display, they need to know how high the rockets will get. The rockets will be aimed toward an empty field and shot at an angle of 35 degrees above the horizontal. The team wants to know how far the rockets will land from the base of the tower so they can fence off the area in advance.

One student, Hilda, says that there is a function $h(t)$ that will give the rocket's height off the ground in terms of the time t elapsed since the launch: $h(t) = 160 + 92t - 16t^2$. Hilda also says that the team can find the horizontal distance the rocket travels with the formula: $d(t) = 92 \cos 35^\circ t$. Again, t is the number of seconds since the rocket was launched, and $d(t)$ is the distance measured in feet.

Help the soccer team find the answers to their questions by completing the following:

1. Draw a sketch of the situation.
2. Write a clear statement of the questions the soccer team wants answered.
3. Describe how you might use Hilda's formulas to help answer the questions stated in Question 2.
4. Using whatever methods you choose, try to get some answers to the questions you stated in Question 2.

The above problem was taken from the textbook: CPM 1:Fireworks Unit